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These papers were presented at the first annual meeting of the American Geophysical Union, by the chairmen of the several sections of the Union. The meeting was held at the National Research Council in Washington, D. C., on April 23, 1920. The American Geophysical Union is also the Committee on Geophysics of the National Research Council and is the National Committee for the United States of the International Geodetic and Geophysical Union.

PRESENT STATUS OF GEODESY AND SOME OF THE PROBLEMS OF THIS BRANCH OF GEOPHYSICS

BY WILLIAM BOWIE

Geodesy is one of the oldest of the geophysical sciences. Originally it embraced only a limited field, consisting of the determination of the shape of the earth and its size and the means by which this was done. But now the subject of geodesy is very closely allied with other matters and there are few geophysical questions that do not, either directly or indirectly, concern the geodesist in the solution of his problems.

Almost every country of the world that pretends to any standing has organized a service for the purpose of carrying on what are called geodetic surveys. As a matter of fact, any survey of the land in which the shape and size of the earth are taken into consideration can be called geodetic; thus, the hydrographic surveys along the coast made for a sailing chart are really geodetic surveys and, similarly, a topographic survey of a large area may be considered to be a geodetic survey; but what I have in mind in speaking of geodetic surveys, are triangulation, base measurements, precise leveling, etc.

By means of base measurements and triangulation, the geographic positions of places are determined which are of immense value in the permanent establishment of boundary lines between nations and political sub-divisions of a nation, for the control of various classes of surveys and maps, and for other engineering purposes.

Practically the whole of Europe has been covered by a system of points whose latitudes and longitudes have been determined by triangulation. This is also the case in Japan and in India. Australia, portions of Africa, and some of the countries of Central and South America have made good starts towards extending triangulation over their areas. In the United States excellent progress has been made in this work. There is triangulation around the entire border and there are many arcs in the interior of the country from which triangulation of precise or lower grades may be extended into every area in which detailed surveying and mapping operations are to be carried on.

It will be of interest to outline briefly the methods for carrying on triangulation or trigonometric surveys.

In the first place a spheroid of reference must be adopted which will approximate the actual shape and size of the earth. Then there must be an initial point whose astronomic latitude and longitude have been observed. Next is needed the azimuth or true bearing of a line of which the station occupied forms one end. After these data are available it is necessary to measure with extreme accuracy the distance between two intervisible points on the earth's surface to serve as a base. After the length of the base is known, additional stations are selected with a view to forming triangles by lines which are clear of obstructions and thus intervisible between their ends. It is, of course, a well known mathematical principle that when a side and the angles of a triangle are known, the other sides can be computed. When these sides have been computed from the base, either one of them can be used as the base for a new triangle. In a similar manner we may extend the computations through a long chain of triangles extending in some cases hundreds of miles across a country.

When the base has been measured and the angles observed and the computations of the triangles have given the lengths of the sides of the triangles, then computations can be made which will give the latitude and

longitude of each point and the bearing of each line based on the data for the initial point, at which the astronomic latitude and longitude were observed and the true bearing determined of one line of the triangulation.

This is the simplest case of a geodetic survey where the whole net work depends on some initial astronomic station.

In practice, we have a much more complicated situation. Geodetic surveys have been started from many widely separated astronomic stations and the triangulation based on each has been finally extended until it meets the triangulation nets based on other astronomic stations. When this has been done, it is always found that the geographic positions computed from one astronomic station will differ from the geographic positions of the same points computed from other astronomic stations. We have a very striking example of this on the island of Porto Rico. The astronomic latitude was determined at Ponce, on the southern coast, and at San Juan on the northern coast of the island. The distance across the island, in a north and south direction, between these stations, is approximately thirty miles. The distance between the two stations, as computed from their observed astronomic positions, differs by about one mile from the distance between the two as determined by triangulation. The triangulation distance is correct within ten feet. The cause of this difference in distance as determined by the two methods is the attraction of the mountain mass, forming the Island of Porto Rico, on the plumb line to which the astronomic observations are referred. To the northward of Porto Rico are the deepest parts of the Atlantic Ocean and just to the southward of the island is a vast depth in the Caribbean sea. It can be readily seen that the effect of this distribution of mass in the oceans and in the island and its base is to draw the plumb line towards the centre of the island. This would throw the zenith on the south shore too far to the south and the zenith at the north shore station too far north. The plumb lines projected against the celestial sphere would therefore include a greater arc than they would under normal conditions. This angle is actually about 56 seconds of arc greater than normal.

In carrying on a geodetic survey, as described above, it is necessary to know accurately the elevations of the base lines and triangulation stations above the so-called plane of reference which is mean sea level. Each line that is determined by the triangulation is referred to sea level, that is, the computed distance between any two stations is the distance, along the geoid or sea level surface, between the points where the plumb lines at the two stations pierce this surface. Accurate elevations are also needed in many phases of surveying, in mapping, and in other engineering operations. They are also essential in many branches of science, notably in meteorology, geology, etc.

In order that elevations may have the maximum value to engineers it is necessary that they all be referred to the same, so-called, plane of

reference. The one which has been almost universally adopted is mean sea level. It may be defined as the surface which would coincide with the surface of the oceans and their tide water branches if the tide producing forces should cease to act and there were no movements of the air and the barometric pressure were uniform. This surface can be determined at any point on the exposed coast by continuous tidal observations for approximately three years. The mean of the hourly readings of the tide record for such a period is practically free from error. If observations were made for a long period of years, it would be found that the average position of the surface of the ocean, for one three-year period, would agree very closely with the position of the surface for any other three-year period. The difference, if any, would be very minute.

It has been assumed by geodesists that, insofar as engineering work is concerned, mean sea level at each place along an open coast is in the same equipotential surface. This, of course, cannot be absolutely true because of different densities in the waters of the oceans, different barometric pressures, trade winds, and possibly other causes, but the deviation of mean sea level at one station from the equipotential surface containing mean sea level as determined at another station is so small as to be negligible in surveying, mapping, and other engineering work, and for practically all scientific purposes.

After the mean sea level surface has been determined, lines of very accurate spirit leveling are extended inland from the fundamental tidal stations. Each country of the world has done more or less of this work and in the United States we have tens of thousands of miles of this grade of leveling.

When a network of triangulation has been extended over a large area and astronomical latitudes, longitudes and azimuths have been observed at many of the stations, differences between the astronomic and geodetic positions of the stations are found. These differences are due to the deflection of the vertical, as was explained in the case of Porto Rico, and to the fact that the spheroid of reference used in computing the geographic positions of the triangulation stations differs from the actual figure of the earth. The former differences, namely, those due to deflections of the vertical, tend to be more or less accidental in character, while those due to an erroneous spheroid of reference are systematic.

After the triangulation and astronomic work have been done over an extensive area, the shape and size of the mean figure of the earth may be computed with a higher degree of accuracy, possibly, than the spheroid of reference on which the triangulation was originally based. For one hundred years or more we have made closer and closer approximations to the true figure of the earth, as the triangulations of the various nations have been extended. In the earlier of these determinations there was considerable uncertainty, owing to the deflection of the vertical caused by

mountain masses and other topographic features. Some of the determinations of the figure of the earth were based on all of the observed geodetic data, that is, triangulation and connected astronomic stations, and other determinations were made to depend only on those data from which stations with large deflections of the vertical had been eliminated.

In the late fifties of the past century, Archdeacon John H. Pratt announced the theory that the geodetic data seemed to indicate that under mountain masses there were deficiencies of matter approximately equal in quantity to the masses that were above sea level. This theory was frequently commented on and studies were made to intepret certain geological features by means of it.

In 1889, Major C. E. Dutton, in an address before the Philosophical Society of Washington, entitled "On some of the greater problems of physical geology,"[1] applied the term "isostasy" to this theory of equal pressure, that the mountains are accompanied by deficiencies of mass under them and that there is an excess of matter under the oceans.

About fifteen years after Dutton gave prominence to this theory, Prof. John F. Hayford, at that time Chief of the Division of Geodesy of the United States Coast and Geodetic Survey, began investigations which showed rather conclusively that the material above sea level is counterbalanced to a marked degree by a deficiency in density of material in the column directly under the exposed material. This column, which is affected by this decreased density, was found by Hayford to extend to a depth of approximately 122 kilometers, if the compensation is assumed to be uniformly distributed. Hayford also found that the column extending approximately 122 kilometers below the bottom of the ocean has a density in excess of normal that will balance approximately the deficiency of matter in the mass of water directly above the column.

This condition of approximate equilibrium in the earth's material at a depth of approximately 122 kilometers was applied by Hayford in determining the figure of the earth from geodetic data in the United States. The results appear in his report published in 1909,[2] which gives dimensions for the earth that are believed to be of a higher order of accuracy than any previously arrived at. This is because the theory of isostasy was considered in this work. It is hoped that in the future the theory of isostasy can be considered in deriving a figure of the earth from all available triangulation data in the world.

In order that this figure of the earth may be computed it will be necessary to have some central office to guide the investigators of the several countries who may assist in the work. Undoubtedly this central office should be under the direction of the Division of Geodesy of the International Geodetic and Geophysical Union, although the central office itself might be located in a geodetic organization of any one of the countries adhering to the union.

In North America great progress has been made in coördinating the geodetic work of the several countries. Some years ago, there was adopted for the triangulation net of the United States what was called the United States Standard Datum. This datum had its designation changed to North American Datum when, in 1913, it was adopted by Canada and Mexico. It is hoped that within a few years a continuous triangulation will be extended into Alaska from the United States and then that territory can also have its triangulation placed on the same datum as the rest of the continent.

A datum may be defined as the adopted latitude and longitude of some station of the triangulation and the azimuth of a line of the triangulation radiating from that station. It also includes the dimensions of the reference spheroid, on which the triangulation is computed.

The advantages of a single datum are, first, that it makes it possible to compute the figure of the earth with greater accuracy and, second, that it avoids the confusion existing in the maps at the frontier of any country when the adjoining countries base their surveys and maps on different datums. Whenever the datums are different, boundary monuments have different geographic positions on the maps of the two countries and there are overlaps, gaps, and offsets in the maps, which frequently result in controversies over the boundary line.

One of the greatest needs in Europe to-day, from a geodetic standpoint, is the adoption of one datum for the whole of its area. This would involve a tremendous amount of revision of the computation and adjustment of the triangulation nets of the several countries, but the resulting benefits would far offset the cost of the work.

Trigonometric leveling is a branch of geodetic work which, though important, is not as much so as triangulation and precise leveling. It is employed in determining the elevations of the triangulation stations in order that the lines observed may be corrected to what they would be if the observations were made at sea level. Theoretically, trigonometric leveling should furnish data from which to compute the shape of the geoid, but this is impracticable owing to our inability to know the exact value of the atmospheric refraction at the instant of observation. Trigonometric leveling is of value in determining the elevations of mountain peaks used as stations or those outside of the main scheme of the triangulation. It is by this method that the elevations of Pikes Peak, Mt. Everest, Mt. McKinley and many others, have been determined.

A branch of geodetic work which has been receiving great attention during the past decade or more is the determination of the intensity of gravity. For a number of years observations were made using reversible pendulums in order to determine the absolute value of gravity. Following this, observations were made with what were called invariable

pendulums approximately one meter in length, to get the relative value of gravity or the difference in gravity between two places, one of which was the base station. These results were not particularly satisfactory because the pendulums were swung in the air with rather poor temperature control and the effect of the flexure of the support due to the swinging pendulum was not considered, or, if it was, was not accurately determined.

About 1885, Von Sterneck devised an apparatus which consisted of a quarter-meter pendulum of the invariable type, swung in a case from which the air could be almost entirely exhausted. With this apparatus, the difference in gravity between two stations could be determined with a high degree of precision. There have been modifications of the Von Sterneck pendulums, but only in minor features, the general principle of the apparatus remaining unchanged until this time.

With the short pendulums of the Von Sterneck type it was possible to establish a station in a very short time and at a very low cost. The result has been a great stimulation of effort in this branch of geodetic work. Most of the countries of the world carrying on geodetic operations have adopted this type of pendulum and many stations have been established. In 1908 there were 47 modern gravity stations in the United States, while to-day there are 276 such stations.

Gravity determinations have a number of uses; one is to furnish a formula by which the value of gravity can be computed for any latitude for use in the physical and chemical sciences; another, to furnish data by which the shape of the earth may be determined; and another, to furnish data for making studies of the theory of isostasy or the deviation from normal densities of matter in the outer portion of the earth.

As a result of the application of the theory of isostasy to the gravity determinations in the United States, the shape of the earth, or rather the amount of its flattening, has been determined by investigations made by the Coast and Geodetic Survey and published in 1917 as Special Publication No. 40 of that Bureau. The reciprocal of the flattening was found to be 297.4. This value is probably the most reliable one in existence to-day. There were also derived from this investigation new constants for the gravity formula for computing the value of gravity at any latitude. This formula was adopted by the U. S. Weather Bureau and by the Smithsonian Institution in the preparation of the latest edition of the Meteorological Tables. In that investigation was also determined the depth of compensation; that is the depth at and below which the materials of the earth are supposed to be in a state of equilibrium. This depth was found to be 96 kilometers, which agrees very closely with the value derived by Hayford in his determination of the figure of the earth at stations in mountainous regions only. It is necessarily true that a depth of compensation can be derived from data in elevated regions with greater certainty than where the land is low. The higher land is liable to be freer

from constant or systematic deviation from normal conditions in the immediate vicinity of a station.

The gravity observations not only give a means of determining whether there are regional deviations from normal densities in the outer portion of the earth under the continents and oceans to cause equilibrium at the depth of compensation, approximately 96 kilometers, but they also afford an opportunity to detect abnormal densities in the materials of the earth which are close to the surface but below sea level. Recent investigations in the United States and in India show a decided relation between the value of gravity and the Cenozoic geologic formation. This formation is approximately 10% lighter than the material in the older geological formations. Pendulum observations may be a means of indicating the approximate depth to which the Cenozoic formation extends at any place.

Gravity observations also indicate a relation between the value of gravity and the Pre-Cambrian formation. In this formation the material is, in general, somewhat heavier than normal, and when a gravity station is established at or near Pre-Cambrian rocks the value of gravity is found in most cases to be greater than normal. It is not improbable that a knowledge of this relation between the Pre-Cambrain formation and the gravity anomaly may make it possible to discover crystalline rocks or others that are extra heavy which may be covered by materials of normal or lighter densities. If this proves to be the case, it may be possible to locate areas underlaid by dense rocks close to the surface of the ground, which may have an economic importance in connection with boring for oil. Much money has been lost by oil prospectors in drilling in regions which have been found by them to be underlaid by crystalline rocks.

It is seen that gravity determinations have much more than a purely geodetic value. The results obtained from them will undoubtedly have a very great effect on the science of geology.

There are geophysical problems that would be considerably cleared up if the value of the intensity of gravity over the oceans could be determined accurately. Such observations would make it possible to determine whether the outer portion of the earth under the ocean is in the same isostatic condition as has been found under the continents. It would seem probable that isostasy obtains to the same extent under the oceans as under the land, but it has not been possible to prove this in the absence of accurate gravimetric determinations on the ocean.

It is true that gravity observations have been made over the ocean, but the apparatus, in each case, lacked the necessary precision to give the value of gravity with a sufficiently high degree of accuracy. It is probable that improvements will be made in the apparatus for determining gravity at sea and when one has been perfected to such an extent that it will give results of adequate accuracy, a long step forward will have been made in geodesy.

The theory of isostasy has some corroborative evidence in the form of a relation between the elevation of the ground and the density of the igneous rocks. Recently Dr. H. S. Washington delivered a paper at a meeting of the Geological Society of Washington in which he showed very conclusively that the higher the elevation at which igneous rocks are found, the smaller the density of such rocks. This tends to confirm the theory of isostasy which postulates the higher the land the less the density in the column under it, down to some indefinite depth called the depth of compensation, approximately 96 kilometers.

It is hoped that these studies by Dr. Washington may be continued by him and by others, for their results will undoubtedly be of great value to geophysicists in general and especially to geologists working in the subject of isostasy.

Another matter that is assuming importance in geodesy is the determination of earth tides. These were studied by the International Geodetic Association, a station having been established for this purpose in Europe some years ago. The apparatus used was of sufficient delicacy to show that there is an earth tide caused by the tide producing forces of the moon and sun, but it could not be considered that the results obtained were entirely satisfactory. This condition has been changed, as a result of the construction of an earth tide apparatus by Prof. A. A. Michelson, of the University of Chicago. This apparatus consists essentially of two tubes placed horizontally in the ground, one in a north and south and the other in an east and west direction. These tubes are partially filled with liquid and the tide producing forces cause an inclination of the liquid surfaces, producing the phenomena of tides. The actual amount of tilting of the surface of the liquid is determined by the interferometer method.

The differences between the tides, as observed in the horizontal tubes, and the theoretical tides that would be produced on the earth, if it were rigid, give the measure of the actual distortion of the earth itself. This apparatus of Prof. Michelson is considered to be entirely satisfactory and it is hoped that it may be used nationally and internationally in determining the value of the earth tides at different parts of the earth and under different geological conditions.

From the result of the earth tides, values can be deduced for some of the physical properties of the material of the earth, such as elasticity and viscosity, at different distances from the surface. A knowledge of these will be of great geophysical value.

Seismology is a geodetic subject in the sense that it throws light on the constitution of the interior of the earth and thus assists in investigations of the shape of the earth. As was said in the first part of this paper, geodesy originally had as its function the determination of the figure of the earth, but in order to ascertain why the figure of the earth is such as

we find it, a knowledge of the physical characteristics of the material composing the earth is essential. Seismology in connection with the investigation of the earth tides should furnish much information regarding the physical properties of the material of the earth at various depths.

A subject that is of interest to many geophysicists as well as to astronomers is the variation of latitude which has been studied for a number of years by the International Geodetic Association. While certain parts of the variation have been found to follow certain laws of change there are still apparent irregularities not yet explained.

From investigations dating back to Euler's time it was at first supposed that the latitude variation, if found, would have a period of about ten months, this being on the supposition of a rigid earth. It is found that the principal part of the polar motion has a period of 14 months. From this fact it is inferred that the earth is not rigid, and from the numerical amount of the lengthening of the period we may draw inferences as to the elastic properties of the earth as a whole; these inferences agree in a general way with what has been found by investigations along other lines. We naturally expect the shifting of the pole to be accompanied by a shifting of the ocean waters responding to the change in the centrifugal force of rotation. This latitude-variation tide has already been studied to some extent without in any way exhausting the subject.

This problem of determining the variation of latitude at observatories is really an astronomic one, although it has a geodetic bearing. The figure of the earth, determined from triangulation and astronomic data, is affected though very slightly by this variation. When it was found desirable to carry on systematic observations on an international scale, there was no astronomic organization with sufficient funds to do the work. The problem was presented to the International Geodetic Association, which had a source of funds not available to the International Astronomic Society. The result was that the International Geodetic Association undertook the work and has carried it on continuously, even during the war, for twenty years or more. The work must not be discontinued nor should there be any break in the observations at the international stations in Italy, Japan and the United States.

Geodesists are interested in the science of volcanology or at least in certain phases of the work in that science which may throw light on the constitution of the material, with densities and temperatures, in the outer portion of the earth, and the cause of volcanic activity. It is believed that these data will supplement the information obtained from the several other lines of investigation such as gravity determinations, earth tides, seismology and variation of latitude.

There is another subject which is of particular importance to geophysicists and, I might say, of very great interest to geodesists. This is what may be called geo-mechanics. It would embrace an investigation to

show the relation of forces to movements of material, the origin of forces, and their causes. It would really combine the subjects of isostasy, earth tides, variation of latitude, seismology, and certain phases of volcanology. The data collected in those branches of geophysics should be considered in connection with geology in order to interpret in the light of present knowledge the various movements that have taken place in the earth. Up to this time, it is believed that these data have not been given due consideration and weight in the study of the earth, its structure, forces, etc.

Some of the subjects that have been discussed above are only incidentally related to geodesy, but the results of investigations in them are certainly of great value and interest to geodesists and it is believed that there should be committees of the Section of Geodesy of the American Geophysical Union to consider these questions and be able to inform the members of the Section at any time as to the progress made in these branches of geophysics. There are several of the subjects which have sections of their own, for instance, those of Seismology and of Volcanology.

The Geodetic Section of the American Geophysical Union will not attempt, nor does it desire, to pre-empt any field of geophysics nor will it encroach on the rights and privileges of any other organization. It will simply attempt to keep informed of what is going on in the various fields of geophysics which are of interest to geodesists and it hopes that it may from time to time give some assistance to other organizations in suggesting problems or lines of research. On the other hand, it stands ready to receive suggestions and help from any other organization dealing in any branch of geophysics.

[1] *Bulletin of the Phila. Soc. of Washington*, 11 (51–64).

[2] Supplemental Investigations of the Figure of the Earth and Isostasy from Measurements in the United States.

THE PROBLEMS OF SEISMOLOGY

BY HARRY FIELDING REID

Although earthquakes have received sporadic attention for some time in the past, their systematic study may almost be said to have begun with Mallet about the middle of the last century; and it was only towards the end of the century that instruments delicate enough to register distinct shocks were developed, making it possible to gain some knowledge of the characteristics of earthquake vibrations and the velocities with which they are propagated through the earth. But, notwithstanding the shortness of the time during which these studies have been in progress, the science has been well outlined and first approximations to the solutions of many of its problems have been obtained. The time has now arrived

for more concentrated and still more systematic attacks, from which important results may be confidently expected.

We can summarize the general state of our knowledge of earthquakes as follows:

Nearly all earthquakes are due to the sudden fracture of the rock of the earth's crust, which has been strained by slow earth movements beyond its strength. Strong vibrations are set up at the fractured surface at the time of the fracture, just as vibrations are set up whenever any solid is broken. The earth consists of solid material, which is necessarily elastic, and therefore these vibrations are transmitted through it as elastic waves. There are two kinds of elastic waves: normal waves, where the movement is in the direction of propagation, and transverse waves, where the movement is at right angles to the direction of propagation. These two kinds of vibration advance with different velocities. Their velocities near the earth's surface are about 7 and 4 kms. per second, respectively; but the deeper they penetrate below the surface the faster they go. Therefore, from the characteristics of elastic waves their paths are curved and concave upwards. These important results have been obtained by a study of the time necessary for the two types of waves to pass from the place of their origin to stations at different distances where delicate instruments are installed, which record the time of their arrival. It may seem remarkable that the paths followed by the rays and the velocity in different parts of the paths could be determined merely from the time required to arrive at a number of places on the earth's surface; but, by the help of some rather abstruse mathematics, it can be done. One interesting conclusion which can be drawn from the passage of the transverse waves through the body of the earth is that the earth is a solid and not a liquid sphere; for transverse waves can be transmitted only by solid substances.

There are other waves which are transmitted along the surface of the earth; they must be started in some way when the body waves, mentioned above, arrive at the surface, but we do not know just how near the origin their starting place is.

In many instances submarine earthquakes give rise to great water waves, which have been known to travel from one side of the Pacific Ocean to the other. It has been the general belief that the first indication of these waves along a coast is marked by the withdrawal of the water; and the great elevated wave follows. Although this order is certainly frequent, it is not general, and in many cases the elevated wave is first to appear.

This is a very summary sketch of what has been learned about earthquakes in the comparatively short time that they have been systematically studied; but it will serve as a basis from which to point out the problems which are now pressing for solution.

For our present purposes we can classify seismological problems into:

A. World-wide.
B. Regional.
C. Theoretical and Auxiliary.

A. World-wide Problems

The noted seismologist, the late John Milne, has estimated that fifty or sixty earthquakes, strong enough to be recorded on seismographs in all parts of the world, occur every year. The study and comparison of the seismograms made at these scattered observatories requires international coöperation, and this will be provided for by the new International Geodetic and Geophysical Union.

The problems facing us are numerous, but perhaps the most important is the accurate determination of the time of the earthquake waves to pass from their origin to points on the earth's surface at various distances from it. We already have a fair knowledge of these times, but need more accurate determinations. This information serves two main purposes: it enables us to determine the origins of earthquakes when they occur in inaccessible regions or under the sea, and thus gives us information of the regions of seismic and therefore of geologic activity; and it enables us to trace out the paths and the velocity of the earthquake waves through the earth, thus throwing light on the physical condition of the earth's interior.

The records of earthquakes at distances of more than about 110° from the origin have proved very unsatisfactory. It is still uncertain whether this is due to the imperfections of our observations or, as Oldham thinks, to the existence of a central core in the earth having different physical properties from the material surrounding it, which materially changes the direction of propagation of the seismic vibrations passing through it, and may even damp some of them out entirely. This very interesting suggestion deserves to be tested.

Professor Wiechert and his collaborators have announced the existence of several shells in the earth where the velocity of propagation of the waves suddenly changes. These results are based on the form of the transmission curve; but, unfortunately, this form is not known with sufficient accuracy to justify the conclusions.

Do earthquake waves travel under the continents and the oceans at a different rate? We can say that the difference is certainly small, but we cannot say that there is no difference. These are some of the problems dependent for their solution on increased accuracy of the transmission curve.

The comprehensive work of the Count de Montessus de Ballore on the distribution of earthquake centers on the earth, has shown that 95% of all earthquakes on land occur in two broad zones, one surrounding the

Pacific Ocean and one passing through the West Indies, the Mediterranean, and the East Indies. These zones are the regions where sediments were accumulated in middle geologic times, known as the Mesozoic Age; after consolidation into rock they have been elevated and in places they now form the highest mountain ranges in the world. De Montessus de Ballore's great work need not be repeated; what is now needed is a detailed study of the distribution of earthquake origins in special regions, and this comes under our third class.

But the distribution of earthquake centers under the seas is still in an embryonic state. A good beginning in the study of this problem has been made by Rudolph, but far more data are needed and they can only be supplied by the records of many widely-spread observatories.

Much study has been given to periodic variations in earthquake activity and its possible relation to variations of latitude, to the positions of the sun and moon, to the seasons, etc. Although this line of investigation has not yielded satisfactory conclusions and, in the opinion of some competent seismologists, is not destined to be fruitful, others believe in its possibilities. If it is to be pursued successfully it is important that a good catalogue of submarine earthquakes be added to the catalogue of those occurring on land. There are a sufficient number of seismological observatories in the United States to supply the data required from this country towards the solution of these world-wide problems, though perhaps a new station somewhere in Texas and another in Montana or Idaho might be of advantage. The effort should be to improve the stations already functioning rather than to increase their number. The examination of many earthquake records has convinced me that not a few observatories are provided with poor clocks, or that insufficient care is given to them. Better time-keeping will greatly increase the accuracy of the records. Some of the best observatories should have a more complete equipment. They should have seismographs of high magnifying powers to catch the very feeble movements, and seismographs of low magnifying power, that the characteristics of the stronger movements may not be lost by too wide a range of the recording point. They should have instruments to record vertical motion, a component that has received far too little attention. I believe there are but two vertical component instruments in this country.

When we consider the important area covered by the Pacific Ocean, we find that there are stations in Japan, Formosa, the Philippines, Australia, New Zealand, Chile, Panama, Mexico, California, and Sitka; and on the islands of Oahu, H. I., Guam, and Samoa. The last station was a branch of the Physical Institute of Göttingen and should not be allowed to fall into desuetude. A station on one of the Aleutian islands would also gather valuable information.

Knott has shown that the greater part of the energy of an earthquake

comes to the earth's surface within a short distance of the epicenter, and 95% within a distance of 90°. When we consider that the earthquake originates at a fault and that the mass movement there may be horizontal, inclined, or vertical, we realize that the energy sent out in different azimuths, and indeed at different vertical angles, may be dependent on the direction of the fault and of the mass movement, but no information has yet been obtained on this subject.

B. Regional Problems

The accurate surveys made by the United States Coast and Geodetic Survey of parts of California before and after the earthquake of 1906, made it very plain that slow movements of the earth's crust during some scores of years finally brought about a rupture of the rock along the San Andreas Fault, and that the two sides of the fault flung back in opposite directions under the elastic forces set up in the rock by those movements. But no information was gained regarding small ruptures taking place preliminary to the main break, nor regarding the location of the many sudden movements after the shock, causing what are known as aftershocks. A methodical study of a limited area where small shocks are frequent and strong shocks are occasional would throw much light on the sequence of events leading up to the rupture producing a strong shock, on the way in which the rupture is gradually enlarged, on the depth to which it extends, and on the order according to which quiet is gradually restored. It seems not impossible that the location of the preliminary shocks might serve to block out the fault along which a rupture was about to take place. Davison's study of the shocks preliminary to the severe Mino-Owari earthquake of Japan in 1891, seems to encourage this idea. But it is important that the actual origins of the shocks should be determined; it is insufficient merely to list the places where the shocks were felt; this last method has led to serious error in the past. By placing simple monuments in properly selected positions and fixing their relative positions and heights from time to time by careful surveys, the slow movements of the crust and the relation of these movements to earthquakes could be discovered. Information could also be gained regarding the relation of the geological structure and the physiography of a region to its seismic activity.

The velocity of transmission of earthquake vibrations in the immediate vicinity of the origin and near the earth's surface, can only be satisfactorily studied by placing a number of seismographs at comparatively short distances apart, and not far from the origin of the shock. A good determination of this velocity would add precision to many seismological problems.

There are many other problems that come in this class. Microseisms, small vibrations continuing for hours, frequently interfere with the satis-

factory recording of earthquakes. The origin of these vibrations is not understood. They may be connected with meteorological conditions. The actual movements of the ground, velocity and direction of propagation of the microseisms should all be studied and compared with the direction of the wind and the direction of the barometric gradient. The concentration of a number of similar seismographs in a limited area would furnish the proper conditions for the solution of this problem.

The influence of the foundation, rock or alluvium, upon the intensity of the vibrations; the relative intensity of the movement in a mine and above ground, due to local or to distant earthquakes, etc., are examples of regional problems before us.

The study of the actual effects of strong earthquakes usually follows somewhat different methods. It requires the examination of the region where the shock has left visible marks by a competent seismologist. The experiences of persons present must be collected; the damage to buildings and other structures noted; and the relation of the earthquake to the geology of the region determined. The slow surface waves in alluvium, sometimes called "gravity" waves, are still a great enigma, and require special attention. A few portable seismographs should be available and should be placed in suitable positions to record and locate the aftershocks. If in addition to these field studies, instrumental records before, during, and after the strong shock should have been made the two kinds of information collected would not only supplement but would add greatly to the value of each other.

C. Theoretical and Auxiliary Problems

The results obtained by world-wide and by regional studies must be woven together into a unified whole and the fundamental laws of the sciences deduced. This kind of work is only done occasionally, and then by individuals who have developed an unusual insight into their science. There are many problems, however, which do not come either under the world-wide or the regional problems, which are important either in helping the solution of these problems, or in elucidating the meaning of the data collected. For instance, the theoretical and experimental study of seismographs is of great importance. It looks to a better interpretation of the seismograms, and to improvements of the instruments, or the development of new forms of instruments. This is work to be done in the study or in the laboratory, but should be in close touch with observations in the field. In particular, we need an instrument that will give a correct record in the shaken area of a strong earthquake.

Then there are many problems, such as the actual genesis of the vibrations, the transmission of body and surface waves, the changes in the character of the waves on reflection, the reasons for differences in the instrumental records of different earthquakes, the peculiarities of earthquake sea-waves, etc.

Although the various seismological problems have been, for convenience, classified into three groups, it must not be inferred that they are sharply defined and distinct; each group will contribute to the problems of the others. Instruments that are primarily intended for world-wide problems will yield information bearing on local problems in their neighborhood; and instruments meant for local problems will also record the world-shaking earthquakes.

Relations to Other Sections of Geophysical Science

For convenience we divide science into branches, influenced largely by differences in the methods used. But nature knows no such dividing lines; and we find that many problems can be solved only by the cooperation of several branches. This is especially true of the geophysical sciences. The special section of seismology looks to all the other sections for help; to geodesy to determine the slow earth movements, horizontal or vertical, that lead up to earthquakes, and the sudden movements that take place at the time of the shock; to meteorology to tell us the atmospheric conditions when microseisms are prevalent; to physical oceanography for information about the deeps of the oceans, along whose borders submarine shocks are common.

The independence of earthquakes and volcanoes has been strongly emphasized during the last thirty years, because nearly all the strong earthquakes were unaccompanied by volcanic phenomena of any kind; and, on the other hand, the great volcanic outbursts, such as Krakatoa in 1883 and Mt. Pelée in 1902, caused only insignificant earth tremors. These facts cannot be contraverted; and still we must not forget that the volcanic belts lie in or near the great earthquake zones; and there may be some common cause of both classes of phenomena.

The influence of earthquakes on suspended magnets may be assigned, with much confidence, merely to the mechanical vibrations. But may not the altered state of strain, in the neighborhood of the generating fault, change the magnetic condition of the rock there if the rock contains magnetic minerals, so that the general magnetic field might be modified? Or may not the passage of violent earthquake waves through magnetic rock alter its magnetization, especially if that magnetization was assumed at some time when the earth's field was different from what it now is?

Seismology does not ask without giving. It can contribute its conclusions to the other branches of science and help them in the solution of their problems.

THE STATUS AND PROBLEMS OF METEOROLOGY

By C. F. Marvin

Meteorology is one of those sciences which developed on the side of practical application for the welfare of mankind to a far greater extent

than the development on the scientific or pure theory side of the problem. As a result, meteorologists responsible for the practical applications to human welfare are at a distinct disadvantage on account of the imperfect and incomplete theoretical development of the science.

The United States was distinctly the pioneer among all nations in officially and formally creating a meteorological service. This was done by a joint resolution of Congress in 1870, which provided that advices, warnings and forecasts concerning future weather conditions should be prepared and issued for the benefit of agriculture, commerce and navigation.

The greatest achievement of the Weather Bureau is doubtless that of creating the present organization of a service of applied meteorology from the nucleus resulting from this early legislation.

Taking the U. S. Weather Bureau as a prominent example of a well-developed, thoroughly organized agency for the application of the principles of meteorology to the welfare of man, it seems appropriate to outline below, in the briefest possible manner, the principal activities and subdivisions through which this public service is rendered.

THE WEATHER BUREAU

A Daily Service of the Science of Meteorology Applied to Human Welfare.—When Congress created the Weather Service in 1870, the primary object was to benefit and protect navigation on the Great Lakes and Atlantic coast by advance warnings of dangerous storms. Later the issue of flood warnings was added to its duties. To-day there is scarcely any important industry or activity of the nation which is not to a greater or less extent influenced by weather conditions, and therefore needs the advices, information and economic benefits which flow from the full and efficient administration of all its duties.

These benefits are of great variety and the information, advices and warnings of the Weather Bureau serve the immediate needs of practically every interest and industry of the nation. Great atmospheric disturbances visit with relative frequency one section of the country or another. Violent storms, cold waves, frosts, freezes, hurricanes, floods, heavy snows, and the like, repeatedly cause destruction of property, such as shipping on the Great Lakes and coastal waters of the nation, and, in recent times, the lives and property engaged in aerial navigation. Crops and farms in flooded districts are laid waste. Lambs, livestock, and meat animals in the stock ranges of the west are killed by blizzards and cold waves. Orchard crops, truck gardens, and vineyards everywhere are damaged by frosts and freezes. By forecasts and warnings, issued and disseminated well in advance, great economic benefits and saving accrue to the nation through the precautionary measures which can be taken on timely ad-

vices to minimize or ward off injuries which otherwise inevitably attend the great atmospheric phenomena mentioned.

The following comprise the principal items of service:

SUMMARY OF WEATHER BUREAU ACTIVITIES

1. Weather Forecasts and Warnings

Weather forecasts and warnings are based upon simultaneous observations of local weather conditions taken daily at 8 a.m. and 8 p.m., 75th Meridian time, at about 200 regular observing stations scattered throughout the United States, and upon similar reports received daily from points in the West Indies, Central America, Hawaiian Islands, Alaska, Canada, Bermuda, and a few other parts of the Northern Hemisphere. The reports are received by telegraph, charted, and the forecasts and warnings deduced therefrom are issued from district centres located at Washington, D. C., Chicago, Ill., New Orleans, La., Denver, Colo., and San Francisco, Calif. The forecasts and warnings are of varying classes as follows:

(*a*) *Daily Forecasts and Weather Information.*—These forecasts are issued twice daily in the a.m. and p.m. and are telegraphed to about 1,600 principal distributing points and thence they are further disseminated by telegraph, telephone, radio and mail to several thousand communities. They are furnished also to the press association and newspapers and are published by practically every daily newspaper in the United States, thus reaching all sections and every class of people. The weather observations on which the forecasts are based are printed in map and bulletin form and mailed or delivered to commercial, marine, agricultural, industrial organizations, educational institutions, etc.

These maps and bulletins are prepared, printed and distributed within three hours after the observations are taken.

The manifold uses to which the daily weather forecasts and weather information are applied are impracticable of enumeration as the life and activity of the whole population is more or less affected by the weather and receives more or less benefit therefrom.

(*b*) *Weekly Forecasts.*—Forecasts are made each Saturday for the six days beginning with the ensuing Monday. These forecasts are made separately for nine districts covering the entire United States, and necessarily are couched in general terms. They are immediately telegraphed to certain designated centres whence they are further disseminated by telegraph, telephone, and mail, and are published in Sunday and Monday newspapers following the day of issue.

These forecasts are intended more particularly for the agricultural interests, as a guide to their activities during the growing season; and the Weather Bureau is in receipt of many communications testifying to their great value.

(*c*) *Shippers' Forecasts.*—Forecasts of temperature expected within the following 24 to 36 hours that will be injurious to perishable products in shipment are made by officials in charge of the stations located in large commercial centres. They are issued only when temperatures of 32 degrees or lower, or 90 degrees or above, are expected.

Special distribution of these forecasts is made to railroads and shippers of perishable products. Shipments are to a large degree regulated by these forecasts. The saving affected thereby is enormous.

(*d*) *Storm and Hurricane Warnings.*—This service consists of the issuance to shipping interests on the Atlantic, Pacific and Gulf coasts and in the waters adjacent thereto and on the Great Lakes, of warnings of the approach of storms dangerous to shipping, and to the coasts of the South Atlantic and Gulf States and adjacent regions of the approach of hurricanes. These warnings are disseminated by means of flags by day and lanterns by night, displayed at about 425 ports. In addition, the warnings are tele-

phoned and telegraphed to practically all maritime interests and broadcasted by wireless to ships in the threatened regions. So nearly perfect has this service become that scarcely a storm of marked danger has occurred for years of which ample warnings have not been issued from 12 to 24 hours in advance.

The sailings of the immense number of vessels engaged in our ocean and lake traffic are largely determined by them. Warnings displayed for a single hurricane are known to have detained in port on our Atlantic coast vessels valued, including their cargoes, at over $30,000,000. Fewer dangerous storms than usual occurred during 1919. However, one unusually severe one occurred on the Lakes in November, accompanied by wind velocities of over 80 miles an hour. Owing to the ample advance warnings not a single marine casualty was reported. Only one tropical hurricane occurred, namely, that of September 6–14, 1919, that passed over southern Florida, traversed the Gulf of Mexico and struck the coast of Texas near Corpus Christi. It was attended by large losses in life and property that were inevitable, but the saving to ships and other property as well as lives was incalculable because of the warnings that were distributed well in advance of the storm. This hurricane was of extraordinary intensity and the efficiency of the warnings was so complete that the portion of the Gulf of Mexico which it traversed was practically cleared of shipping; otherwise many ships valued at millions of dollars would have been caught in the storm. Few, if any, ships caught within the storm centre could have survived. This efficiency of service added to the difficulties of the forecaster, because he was unable to secure wireless reports from the region of the storm centre which were so necessary for an accurate determination of the hurricane's direction and progress.

(*e*) *Cold Wave Warnings.*—These are warnings of sudden and destructive temperature changes that frequently sweep across the country during the winter season. They are issued from 24 to 36 hours in advance and are disseminated throughout the threatened regions by means of flag displays, by telegraph, telephone and mail.

The uses made of these warnings are manifold. They are especially valuable to railroads in the making up of trains and the maintenance of schedules, to shippers of perishable goods, cattle growers, and innumerable business organizations. The warnings issued in a previous year for a single cold wave of exceptional severity and extent resulted, according to reliable reports, in a saving of over three and a half million dollars. No unusually severe cold waves occurred during 1919, although warnings were issued on about 30 different occasions for moderate cold waves.

(*f*) *Frost Warnings for Truckers, Tobacco, Cranberry and Sugar Growers.*—These warnings are issued whenever conditions indicate the occurrence of damaging frosts. The growers take advantage of these warnings to protect their crops by the various means in vogue. In many instances the growers are dependent upon the Weather Bureau for these warnings as without them their industries would be too hazardous to be profitable. This is especially true of early trucking in the South Atlantic and Gulf States. During the year 1919 rather fewer instances than usual of weather conditions necessitating these warnings occurred; but during the year previous it is of record that warnings of a damaging freeze saved nearly one-third of the entire sugar crop in Texas and Louisiana by enabling the growers to windrow the standing cane.

Special frost warnings are sent to all parts of the tobacco-growing section in Wisconsin and Connecticut. Information of current weather conditions during the growing season is also supplied to the other large producing areas.

Frost warnings are sent to the principal cranberry-growing sections of the country, which enable growers to protect their crops by flooding or other protective means. They are indispensable to this industry.

(*g*) *Fire Weather Warnings.*—These warnings are issued whenever conditions are favorable for hot, dry winds, which are favorable to the inception and spread of fires

in the National and other forested regions. They are distributed principally to Forest Service officials and others charged with forest protection.

The warnings aid in the protection of the forests by enabling crews and apparatus to be ready for action. Numerous testimonials of their value have been received. several in the form of resolutions passed at the conventions of Forestry Associations, urging appropriations.

(*h*) *Forecasts and Warnings for Stock Growers.*—This service covers the western and northwestern portions of the country and consists of warnings of cold waves, high winds and heavy snows. These warnings enable stock men to make proper provision for the protection of stock exposed on the ranges. They are of especial value to sheep men during the shearing and lambing periods as heavy losses sometimes occur when proper protection has not been provided, sometimes as much as 50 per cent of individual flocks. In the States of Washington, Oregon, and Idaho alone there are two million sheep valued at $12,000,000, exposed to the weather and the sheep men depend largely on the Weather Bureau for information as to when protective measures should be taken.

(*i*) *Fruit Frost Warnings.*—This service is maintained in a number of important fruit growing sections of the country, particularly in California and Oregon. A killing frost without warnings or protective measures means disaster to the crops. So important has the work become that a trained meteorologist is assigned to important districts during the frost danger season to determine how low the temperature will fall and to advise as to the extent of the protective measures that must be taken.

(*j*) *Forecasts for Alfalfa Growers.*—Special warnings are issued for alfalfa seed growers in the principal seed producing sections of the West, and for alfalfa hay making in nearly all sections from the Mississippi Valley westward. Frosts are frequent at about the time alfalfa seed is maturing and are very damaging to unharvested crops. It is estimated that near harvest time, the crop increases in value at the rate of about five dollars an acre during each twenty-four hours; if it is cut too early, much loss is sustained but a heavier loss may result from frost if not cut early enough. On the receipt of a warning, the growers vigorously push harvesting operations to save as much seed as possible.

In addition to the seed growing service, fair weather warnings are sent to all the principal alfalfa growing districts of the West, during the hay-harvesting period. In Oklahoma alone two thousand alfalfa growers receive these special forecasts which are distributed through the County Agents of the Farm Extension Service.

(*k*) *Fruit Spraying Forecasts.*—A special fruit spraying service is maintained in New York and other important fruit growing districts, by which special forecasts are made for the benefit of fruit growers during the spraying season. The effectiveness of spraying depends largely upon the weather conditions at the time and immediately after spraying, and this service is of much value in supplying information as to the best time at which this work should be done. This service has been so successful that requests are on file for the establishment of similar work in a number of districts not now covered.

2. HIGHWAY WEATHER SERVICE

This service consists of the issue of bulletins, daily, semi-weekly, or weekly, depending on locality, giving the conditions of the main highways in the section in which the distribution centre is located, and their passability as affected by rains, heavy snows and other conditions. Detours and other pertinent information are included. Bulletins are issued at about 50 stations in 30 states. These bulletins are used by travelers in determining journeys and routes, in the dispatching of truck, determining loads, etc.

3. Aerological Investigations; Forecasts and Warnings in Aid of Aviation

This work involves the obtaining of free air data in different parts of the United States for the purpose of making these data immediately available for the information and assistance of the forecasters of the bureau, army, navy, and post office fliers. The observations also are summarized, studied, and the data published for the purpose of furnishing reliable information as to free air conditions for the benefit of aviation in general and especially for the aviation and artillery services of the army and navy.

Advices to the Navy crews, for the trip from Newfoundland to the Azores, were based to a considerable extent upon free air observations over the ocean. The N. C. 4 reached her destination and the other two landed close enough to the Azores so that their crews were rescued. Without the wind assistance as predicted none of these seaplanes could have flown as far, and the crews would probably have perished.

The British dirigible R. 34 left this country for England just in time to prevent being swept away by a high wind. The forecaster based his warning partly upon upper air observations. Its officers have stated that had not the warning been received the air ship, valued at more than $1,000,000, would have been lost.

The N. C. 4's recruiting trip during the autumn was successful from beginning to end largely because her pilot followed the weather forecasts, which were based partly on upper air observations.

There is no authenticated instance in which an aviator has had an accident due to bad weather, when favorable weather had been predicted.

4. Marine Meteorology in Aid of Navigation; Vessel Reporting

Meteorological observations taken on ships plying all the seas of the world are collected, charged, and discussed, and the information furnished to the Hydrographic Office of the Navy and published in the pilot chart issued by that office. The information is also made available for vessel owners, ship builders, and others engaged in work relating to ocean navigation. Moreover, the information is of value and assistance to the forecasters in preparing weather forecasts, storm, hurricane and other warnings. A full understanding of the meteorology of the ocean is of tremendous economic value in determining the safest and best voyage routes, the location of dangerous storm areas and meteorological facts of value to mariners.

Vessel reporting stations are maintained at Cape Henry, Va., Sand Key, Fla., Point Reys, Calif., and North Head, Neah Bay, Tatoosh and Port Angeles, Washington, for the purpose of reporting to ship owners the passage of vessels in and out of straits or capes at a distance from the port of destination. This gives owners several hours advance notice of the docking of ships and enables them to assemble stevedores and make other arrangements regarding loading and unloading, etc.

5. Evaporation and Run-off Investigations

Evaporation investigations are conducted in certain arid and semi-arid regions of the west for the purpose of determining the loss of storage water by evaporation. These results are of direct value to engineers in planning city water supply systems and water and irrigation reservoirs.

A special project is being conducted at Wagon Wheel Gap, Colorado, in coöperation with the Forest Service, which involves the study of the effect of forest covers on stream flow, erosion, and run-off. The scheme involves the making of continuous observations in two nearly similar water sheds in their natural forested condition for a term of years; then to denude one of the water sheds and continue the observations for a similar period of years for comparative purposes.

6. River and Flood Work

This is one of the most valuable and important activities of the Weather Bureau. Service is maintained on all of the navigable rivers of the United States, their tributaries and water sheds. Gages are located at suitable intervals on the rivers for determining water stages and rain gages distributed throughout the water sheds. Stage readings and precipitation are telegraphed daily to district centers where the observations are tabulated, summarized and published on the daily weather maps or in bulletin form. When conditions warrant, warnings of floods are disseminated to all affected interests in the flood areas by telegraph, telephone and other means.

The saving which results from this service consists in transferring goods, stores and other movable property to places of safety. In the south and southwest live stock is pastured in river bottoms many of which are subject to overflow in time of flood. A notice of the coming of the flood permits the owner of the live stock to drive it out of the bottoms, and to safeguard any other movable property that may have been left in the threatened region. Forecasts of extreme low river stages are also of much value to navigation in many rivers.

7. Mountain Snowfall and Water Supply Investigations

This service is maintained in the western states in order to give dependable information relative to the seasonal snow cover, its depth and water content. This information is valuable in determining the probable water supply that will be available both for irrigation and power purposes in those states having little or no rainfall in the summer months.

8. Climatological Work

This work consists of the collecting, compiling, charting, and discussing climatological data of the United States, its adjacent oceans and outlying territories. In addition to the observations taken at the more than 200 regular stations of the Weather Bureau, observations of temperature, weather, precipitation, etc., are taken by cooperative observers located in practically every county of the United States. These observations are compiled and printed monthly and disseminated to the public, to commercial exchanges, farmers, engineers and home seekers and many other classes. The value of this work is incalculable. It affects and benefits the entire people; is used in the settling of damage claims, cases at law and for other purposes too numerous to mention. In New York City alone the weather records are brought into court by personal appearance of a Weather Bureau official more than 500 times a year. Several thousand certificates are issued annually over the seal and signature of the Secretary of Agriculture for court use. More than 100,000 people annually make inquiry and are furnished statements regarding past weather records. The economic value of the climatological work of the bureau is enormous.

9. Agricultural Meteorology

This service consists in the collection and dissemination of information relative to the effect of current weather conditions on farm activities, the growth of vegetation and the development of crops and their advancement as compared with an average season. Advice is given and information disseminated relative to the utilization of the special forecasts and warnings issued by the Weather Bureau in the interest of various lines of agriculture and horticulture and stock raising. A number of lines of investigations are being conducted as to the relationship between weather conditions at various periods of the growing season and crop yields.

(*a*) *Weekly Weather and Crop Bulletins.*—The National Weather and Crop Bulletin issued weekly at Washington contains a detailed summary of the weather conditions throughout the country for the period covered by each issue and indicates the resulting

effects on the development of vegetation and farm activities in general. In addition, the Bulletin contains from time to time charts and discussions bearing on the relation of climate to crops and agricultural operations.

The value of this publication is evidenced by the popular demand for it, which has grown to such proportions as to severely tax the printing facilities of the Bureau. There are about nine hundred paid subscriptions to this publication and about twenty-seven hundred other recipients.

In addition, a weather and crop summary is issued during the active agricultural season at each state center, forty-two in number, which is disseminated by bulletins and through the public press.

These services are of much value to those interested in agricultural operations throughout the country, as they afford detailed information as to the prevailing weather in all sections and its effect on farming operations and crop development.

(*b*) *Corn and Wheat Service.*—This service is maintained during the active growing season of each year, April 1 to September 30, in sixteen principal grain states. Daily weather reports are received from one hundred and eighty-seven special reporting stations. Daily bulletins are issued from nineteen points. A weekly corn and wheat region bulletin is issued also at the regional center, Chicago, Ill.

This service is of especial benefit to farmers, dealers, millers and others interested in the cereal crops of the United States, and gives accurate information as to prevailing weather conditions throughout the principal producing areas during the period of crop development. There are constant and persistent demands for extensions of the service into regions not now covered.

(*c*) *Cotton Region Service.*—This service is similar in organization and operation to the corn and wheat region service, and covers the cotton growing states of the country. About two hundred reporting stations are maintained. Bulletins are issued from 26 stations located in the cotton belt. The service is maintained in the interest of cotton growers and others engaged in various lines connected with the industry. It is considered by cotton growers and dealers to be of indispensable value.

(*d*) *Cattle Region Service.*—This service is maintained for the collection and dissemination of information of current weather conditions over the great grazing districts of the west and southwest as affecting the stock industry. Weekly bulletins are issued from a number of central points throughout the year, giving information as to rainfall, temperature, snowfall, and general grazing conditions on the ranges.

10. Solar Radiation Investigations

The importance of sunshine as a weather and climatological factor is fully recognized by meteorologists, climatologists, biologists and others. Until a comparatively recent time, only the duration of sunshine has been recorded. The object of the solar radiation investigations is to measure the intensity of the sun. These data have not only direct application to agriculture, but have a value in many other ways. Illuminating engineers have for a long time been asking for measurements of daylight or sunshine intensity expressed in light units. This information is of special value to them in connection with the designing of factories and other buildings to meet the requirements of state laws relative to natural light.

11. Seismological Investigations

The object of these investigations is to determine how frequently on the average and how severely the various portions of the country are shaken by earthquakes. From the data thus collected, it is determined what sections of the country are seismic; just where the active cracks are, and their liability to further slips or movements. This information is of prime importance in enabling engineers to avoid the placing of aqueducts for city water supplies, dams and other structures on or across fault lines.

12. VOLCANOLOGICAL INVESTIGATIONS

This work is now confined to studies of the activity of the Kilauea Volcano located on the Island of Hawaii. It has been conducted by the Weather Bureau for little more than a year, is scientific in character, and it is impracticable at this time to assign thereto any pecuniary return to the people.

APPROPRIATION

The appropriation for the Weather Bureau for the fiscal year ending June 30, 1920, amounted to $1,880,200, almost the whole of which is expended for public service, and under existing conditions but little is available for investigation and research. It is very easy to show that benefits accrue to the United States in the proportion of much more than one thousand to one.

PROBLEMS OF METEOROLOGY

It is now necessary to carry on all the practical routine functions of this service while the forecasters and officials are still doubtful concerning many of the most fundamental and important laws and causes of the atmospheric phenomena with which the science deals.

Education.—Very few of the great universities carry any courses in advanced meteorology. In fact, meteorology receives far less attention in the institutions of learning than is given to almost any other branch of science. Tendencies to the more profound recognition of meteorology in education are in evidence, and it is earnestly hoped that much more intensive attention can be given to this science in view of its enormous practical importance in the everyday life of humanity.

Meteorological Laboratories.—Progress in other lines of science has been made in many cases by leaps and bounds through the establishment of laboratories for the conduct of specific investigations. The really great problems of meteorology cannot be brought within the domain of a laboratory, and no real meteorological laboratory yet exists, although certain elemental phenomena of meteorology admit of investigation in the best physical laboratories. Progress is delayed, no doubt, because of the inability to subject the great major phenomena of meteorology to laboratory and controlled investigation. The difficulties involved in these considerations appear to be inherent and insurmountable, leaving to the student only the alternative of minutely analyzing the complex mass of meteorological data, with the view to working out the more or less indefinite correlations and the coefficients of involved equations representing all the factors in operation in arriving at any particular result. The masses of data available are enormous, and meteorology has been criticized for continuing to accumulate, as it must from other important considerations, observations of every sort of atmospheric condition while its discussion is sadly neglected. Students are needed to study this mass of data, and it is often difficult to know where to begin or how to proceed.

Meteorology of the Oceans.—While the mass of data available for land and continental areas is very great, the details of information available from the larger expanses of ocean areas are correspondingly meager and inadequate. Owing to the uniformity of conditions over the ocean areas, as well as their tremendous extent, we should expect the sequence of atmospheric phenomena to be more simple and easy of investigation. The collection of more complete data from ocean areas and its intimate analysis and discussion are, therefore, most important.

Aerology.—The term *aerology* has come recently to be applied to the meteorology of the free atmosphere in contrast to observational data dealing specifically with surface conditions. The appliances and agencies for conducting observations in the free air have been improved, multiplied and made highly practical, and the attainment of practical navigation of the air has not only added to the observational possibilities but has imposed new and important obligations on the meteorologist to make applications of his science to the aid of aerial navigation. Free air observations over the oceans especially are greatly desired, although very difficult to secure. The whole field of investigations in the free air is relatively only recently occupied and appears to be fruitful of the most important developments. The organization of systematic studies of upper air observations to be carried on concurrently with the making of the observations is at once among the foremost problems awaiting consideration.

WORLD METEOROLOGY

The development of the so-called world meteorology as typified by the collection, summarization, and publication of simultaneous weather reports from all civilized countries has been the goal of meteorologists for many years. The desires of a number of international meteorological conferences in pre-war days are being realized by the publication the "Réseau Mondial" by the Meteorological Office of England. The "Réseau Mondial," however, provides only for monthly summaries of pressure and temperature.

In 1914 the Weather Bureau began publication of a daily chart of pressure and temperature of the Northern Hemisphere from telegraphic and cable reports. The machinery for the collection of the data was put out of commission during the war and has not yet been restored.

In considering the resumption of this undertaking it may be viewed from a totally different angle than formerly, viz., is it not possible by sacrificing the time element to prepare a daily pressure and temperature map of the Northern Hemisphere—say 30 to 90 days after the current dates have elapsed, such maps to serve for study purposes only and to be reproduced in a small edition?

Closely coupled with the above proposition is the need already mentioned of more frequent meteorological observations over the oceans,

more especially the Pacific. In this connection reference may be made to the Committee of Exploration of the National Academy of Sciences which has had under consideration for several years plans for the exploration of the atmosphere over the Pacific. What I have more particularly in mind, however, is the necessity for more meteorological observations from vessels navigating the Pacific. The ordinary means at our command that can be applied to stimulating interest in meteorological observations by vessel masters may not be sufficient to procure an immediate response to this need. In any event it will be helpful and necessary to secure the active coöperation of all the great dominating interests concerned in the development of the Pacific area.

UNIFORMITY IN PUBLICATION

The one great handicap to the use of the accumulated meteorological data is the fact that adequate summaries for representative stations in different parts of the globe are not available. An important forward step will have been taken when the various meteorological services shall agree to compile according to some uniform plan and publish monthly and annual summaries for representative stations under their jurisdiction. This in effect would be an extension of the data now published by the English Meteorological Office to include other elements than pressure and temperature. The subject is one for international conference and consideration; meanwhile each country should begin the preparation of a digest of its own data for the needs of its own citizens.

TERRESTRIAL WEATHER AND SOLAR ACTIVITIES

A profound understanding of the problems of meteorology requires a correct discernment of the ultimate causes of weather. In recent years much attention has been directed to the establishment of detailed correlations between solar activities and terrestrial weather. It seems most important to proceed cautiously in a matter of this kind, since it would appear that the views being advocated by a few leaders in these investigations must be regarded with skepticism and distrust in the forms in which they have thus far been presented. All are in accord that the sun is the ultimate source of all our atmospheric phenomena commonly designated weather. It seems that the daily sequence of sunshine and darkness; the varied distribution of clear and cloudy skies; diversities of surface cover added to contacts of land and water areas, including the phenomena of evaporation, condensation, and precipitation; the cycle of the seasons, and above all the fluctuating but nevertheless perpetual contrasts of surface temperatures, ranging all the way from the heat of the tropics to the intense cold of the polar zones constitute a complex series of varied and changeable influences seemingly abundantly adequate to cause and explain every feature of our weather conditions, however changeable we may find them.

These differences and contrasts on the one hand perpetually disturb the orderly arrangement of air densities and pressures demanded by gravity. The latter, on the other hand, as perpetually and continuously sets portions of the air in motion, in order to establish and maintain a state of equilibrium, which, however, is never attained. We must clearly recognize that *the ceaseless complex changes in and motions of our atmosphere* represent in fact the only state of equilibrium possible between gravity on the one hand and solar heating of the earth on the other.

The disposition on the part of some to attribute important phenomena of terrestrial weather to minor solar activities is believed to be a mistake. It is in fact believed that apparent changes in solar activity, especially of the thermal character, are in reality altogether of terrestrial origin and that the imaginary correlations between terrestrial and solar activities are really the simple relations between atmospheric phenomena at one point and possibly those at another on the earth itself. It is most important that these major problems of cause-and-effect relation between solar and terrestrial activities should be observed, not only with the greatest possible accuracy but that the observational data themselves should be thoroughly analyzed and discussed, first, to establish the magnitude of the errors of the observation, and second, to determine the possible relations between solar and atmospheric effects.

Seemingly the greatest need in meteorology is that of a master mind to direct itself comprehendingly and intensively to the great problems which the science still presents.

SOME OF THE CHIEF PROBLEMS IN TERRESTRIAL MAGNETISM AND ELECTRICITY

BY LOUIS A. BAUER

1. *Analysis of the Earth's General Magnetic Field:* The early completion of the general magnetic survey of the globe, as undertaken by the Department of Terrestrial Magnetism of the Carnegie Institution of Washington and coöperating countries, will afford the necessary data for the investigation of some of the so-called "greater problems of the earth's magnetism." One of these is the determination of the various systems of magnetic and electric forces which together make up the total terrestrial magnetic field as observed at the earth's surface. It is known from previous analyses that the earth's magnetic field consists of an internal system of forces, which constitutes by far the major portion; secondly, an external system, supposedly to be ascribed to electric currents circulating in the earth's atmosphere; and, thirdly, a system possibly such as that of vertical electric currents which pass from the atmosphere into the earth and vice versa. The definite settlement of the question as to the existence of the third system is regarded by Sir Arthur Schuster as one

of the chief outstanding problems in terrestrial magnetism. This problem involves the question whether or not the magnetic forces of the earth can be entirely referred to a potential. If not, then the question arises as to the existence of vertical electric currents, which will be of interest both to students of the earth's magnetism and of the earth's electricity, using the latter term in its most general sense so as to include electric phenomena arising from systems both below and above the earth's surface. The ocean work of the Department of Terrestrial Magnetism has been arranged with the special view of obtaining the requisite data for the solution of this interesting question. But for the war all the necessary data and the results of their discussion would be available now.

2. *Analysis of Continental and Regional Magnetic Fields:* The detailed magnetic survey of the United States—the largest land area over which magnetic surveys with the requisite accuracy and detail have been made—affords further opportunity for the investigation of the problems referred to in the previous paragraph, at least so far as the investigation of those problems can be undertaken for a limited region of the globe. In the respect that the magnetic constants of the earth's general field depend in a very large measure upon the area embraced in the investigations, problems of terrestrial magnetism differ fundamentally from those of geodesy. Concerning this point, a preliminary study was made by the writer in connection with a brief discussion of "Some Results of the Magnetic Survey of the United States" printed in *Science*, May 22, 1908, (812–816).

It was found that the magnetic forces observed at any given point in the United States are the resultant effects of (1) a general or terrestrial magnetic field arising from the general magnetic condition of the earth, (2) a general, terrestrial disturbing cause which distorts the general magnetic condition of the earth at the place of observation, related possibly in some manner to the rotation of the earth, (3) a continental disturbing effect, arising largely from that portion of the North American continent above the general ocean bed, (4) a regional disturbance caused by low-lying magnetized masses or earth-currents covering a certain region, and (5), a local disturbance to be referred to magnetized masses or currents in the immediate vicinity.

A mathematical formula of limited spherical harmonic terms based upon the entire earth cannot, of course, include also disturbances of such restricted areas as are embraced under Nos. 4 and 5, namely, regional and local disturbances, but it appears that such a formula cannot adequately represent even the *continental* effects. In the spherical-harmonic formulae the terms of the various orders advance and recur by trigonometric functions of multiples of the longitude and the latitude. Thus a term involving six times the longitude is supposed to continue around the earth six times in unaltered manner. However, such a term arising, for example, from the North American continental shelf, which in the

United States has a width in longitude of about 60°, would not continue around the earth but would in all probability die out before the European or the Asiatic continent is reached. In brief, the higher harmonic terms of the mathematical formula introduced by Gauss and based on the potential theory, could at the best but simulate or "counterfeit" the facts of nature and not represent the actual truth.

Thus the question arises how far it is really worth while to proceed in the establishment of such a theoretical formula. Although the mathematical expressions may be extended so as to include forty-eight or more unknowns, it has been found that the computed results, magnetic declinations, for example, depart so much from the actual observed results—several degrees—that they could not be used even for purely practical purposes such as those of the surveyor and the navigator. The conclusion therefore reached was that it may be found sufficient for many purposes to restrict the theoretical formulae to a certain finite number of terms involving a limited number of unknowns which represent, from a physical standpoint, the chief and principal facts of the magnetic condition of the earth, as resulting, for example, from system (1) mentioned above, or perhaps better, systems (1) and (2). The magnetic field represented by this limited expression might be called the normal, or reference field, just as the geodesist, who restricts the constants to be determined for the figure of the earth to two or three, calls his adopted figure, the "spheroid of reference." Residuals from the magnetic field of reference would then receive separate or special treatment in accordance with their extent and their character.

Respecting the existence of possible vertical currents, the preliminary analysis of the data of the magnetic survey of the United States indicated a definite system, related in some manner to the general atmospheric circulation and with an average current strength of about $^1/_{30}$ ampere per square kilometer, just as the writer had found in his previous investigations pertaining to the entire earth.[1] The additional data acquired in the United States since the preliminary analysis, will make possible a more complete investigation of this important matter. Of special interest in this connection will be found the quotations regarding Dr. Wilson's investigations (see paragraph 4).

We see then that the data obtained by the United States Coast and Geodetic Survey in the prosecution of the magnetic survey of the United States afford opportunities for interesting and fruitful research of various kinds. There may be mentioned, for example, the correlation between magnetically disturbed areas and geological formations or local deviations of the plumb-line, in which Section *a* (Geodesy) is also interested. Possibly also the data obtained in the United States may throw some light ultimately upon the problem as to the variation of the magnetic elements with altitude, and upon the question as to the effect of local or regional disturbances upon the secular changes of the earth's magnetism.

3. *Effect of Altitude upon the Earth's Magnetic and Electric Elements:* One of the important problems, both from a practical and a theoretical standpoint, is that of making magnetic and electric observations in the upper regions of the atmosphere. A number of investigations of extreme interest fall under this head. How the magnetic data for the United States may contribute towards the solution of some of the problems has already been stated. The preliminary analysis mentioned gave some indication of dependence of vertical-intensity residuals upon altitude. A joint investigation by the Department of Terrestrial Magnetism and the United States Coast and Geodetic Survey was made of the mountainous region about Pike's Peak, Colorado, directly after the eclipse observations of June 8, 1918. The magnetic stations ranged in altitude from about 6,000 feet to 14,000 feet. Whether any definite result may be derived from such mountain observations as to rate of variation of magnetic elements with altitude, in view of the minuteness of the probable variations, depends upon the successful elimination of the local disturbances caused by magnetic rocks. It was found that the region investigated was not wholly free from such disturbances. It will be necessary for the solution of the problem to devise instruments of sufficient sensitiveness and stability to make possible magnetic observations of the required accuracy in air ships. To show the necessity of altitude magnetic observations, it will suffice to say that an infinite number of distributions of magnetic and electric systems within the earth may be found which will satisfy observations made simply on the surface of the earth.

Complete atmospheric-electric and radioactive observations in the upper regions of the atmosphere are of fundamental importance to the problems of the earth's electric field: of its maintenance, and of its variations. It is very much to be hoped that such observations may soon be systematically carried out in the United States.

4. *Variations in the Earth's Magnetic and Electric Phenomena:* The study of these variations opens up a wide field of research of interest not only to the Section of Terrestrial Magnetism and Electricity but also to other sections of the American Geophysical Union. In investigations as to a correspondence between manifestations of the earth's magnetic, or electric, activity with the Sun's activity, one of the fundamental questions is what to take as the complete measure of activity, either for the earth or for the sun. This question also enters into certain meteorological investigations. In this way, and as appears later, some of the problems of Section *d* (Terrestrial Magnetism and Electricity) also touch upon some of those of Section *c* (Meteorology). The following quotations from the Annual Report of the University of Cambridge for 1919, relating to investigations made by Dr. C. T. R. Wilson at the Solar Physics Observatory, are of particular interest here:

"The results obtained in these investigations have suggested a theory, which both accounts for many of the more important phenomena of thunderstorms, and relates them to those of fine-weather atmospheric electricity and terrestrial magnetism....

"The order of magnitude of the currents in thunderstorms, as indicated by the results of observations of lightning discharges, is such as to suggest the possibility of correlating the phenomena of atmospheric electricity with those of terrestrial magnetism."

5. *Magnetic Storms, Polar Lights and Earth Currents:* Many problems of interest to several sciences come under this head. The various pronounced manifestations of solar activity and terrestrial magnetic and electric activity during the past month, especially March 22–23, have called renewed attention to these problems. The magnetic disturbance of March 22–23 was world-wide and was one of the severest recorded. On the evening of March 22 one of the most remarkable displays of the Aurora Borealis was witnessed at Washington and at points in even lower latitudes; at a corresponding time there was a fine display of the Aurora Australis at the magnetic observatory of the Department of Terrestrial Magnetism at Watheroo, Western Australia. During the magnetic disturbances and auroral displays the generated earth-currents caused interruptions on cable and telegraph lines. The sun-spot activity during this period was of a very marked and interesting character. Dr. Abbot reported that the solar constant, as observed by the Smithsonian Institution party at Calama, Chile, on March 23, had reached the low value of 1.866, whereas from January 1 to March 22 it had varied from 1.93 to 2.00; on March 24, the value was 1.905, and thereafter it went up again. This case illustrates well the joint interest of various sciences.

The general effect of magnetic disturbances is to diminish for a time the intensity of magnetization of the earth's field, in severe cases as much as ten percent. It took about three months after the severest magnetic storm on record, the one of September 26, 1909, before the earth's magnetic intensity had returned to its approximate normal value. An interesting theory[2] of magnetic storms has recently been advanced by Prof. F. A. Lindemann, director of the Department of Physics at the Univeristy of Oxford. To overcome some of the difficulties of Dr. Chapman's theory[3] of a stream of charged particles, Lindemann suggests the alternative of clouds of ionized gas ejected from the Sun and driven away by light-pressure. Such a cloud would be completely ionized, and therefore invisible; it would be kept generally together because it contains both positive and negative ions; and yet it would expand sufficiently to satisfy observed facts about the duration of magnetic storms as the earth passes through the cloud—in our upper atmosphere the ions would recombine and cause the storm. The theory is worked out quantitatively and found to satisfy known facts.

6. *Magnetic and Electric Observations During Solar Eclipses:* Intensive observations under this head have been made, especially since the total

solar eclipse of May 28, 1900, which occurred in the southeastern part of the United States, the writer being then in charge of the magnetic work of the Coast and Geodetic Survey. Nearly every prominent solar eclipse since then has been taken advantage of and coöperative observations over the entire globe have generally been made in accordance with a program outlined by the Department of Terrestrial Magnetism. Briefly stated, the general result has been that an appreciable magnetic effect, though of a minute order, is recorded during the period of a solar eclipse, similar in character to the variation experienced by the earth during a solar day, and differing from it only in magnitude. It is believed that the detection and study of an eclipse magnetic effect will be of great importance to theories of the earth's magnetic field and of its variations.

During the famous solar eclipse of May 29, 1919, the Department of Terrestrial Magnetism had two expeditions in the belt of totality: at Sobral, Brazil, under the charge of D. M. Wise, assisted by A. Thomson; and at Cape Palmas, Liberia, under the writer's charge, assisted by H. F. Johnston. At Cape Palmas there was experienced the longest totality (6 minutes, 33 seconds) enjoyed by any of the eclipse parties. The geophysical observations at the two stations will have an important bearing upon the complete interpretation of the light deflections observed by the British expeditions stationed at Sobral, Brazil, and the Île of Principe.[4]

Besides the magnetic results, interesting electric results were also obtained at Sobral, which, briefly stated, are:[5]

a. The potential gradient showed a well-formed minimum beginning with totality and extending until about 20 minutes after totality; the values observed during this period were about 20% lower than the mean derived from two equal periods immediately preceding and following it. *b*. During the period of potential-gradient minimum the fluctuations of the gradient were very much smaller than during the similar periods preceding and following. *c*. The positive and negative conductivities (and, therefore, the total conductivity also), each showed an increase of the order of 20%, beginning just after totality and continuing for about 15 minutes. *d*. The air-earth current density (product of simultaneous values of potential gradient and total conductivity) showed a greater constancy during the period in question than for any equal period throughout the forenoon of the day of the eclipse.

The results for May 29, 1919, at Sobral, are in general agreement with those obtained at Lakin, during the eclipse of June 8, 1918,[6] notwithstanding the great difference between the two stations as regards latitude, elevation, general topography, and distance from sea.

7. *Terrestrial Magnetism and Seismology:* Records of seismic disturbances are frequently obtained on magnetograms and thus arise problems of joint interest with the Section of Seismology. Prof. Reid has made a special study of these magnetograph records and finds that the oscillations

of the magnets producing the records are chiefly mechanical in their origin.[7]

8. *Terrestrial Magnetism and Electricity, and Physical Oceanography:* The problems of joint interest to the two sections relate to the magnetic and electric observations over the oceans, the delineation of disturbed regions, especially near ports, the effect of ocean depths on changes of the magnetic elements as observed over the surface, etc.

9. *Terrestrial Magnetism and Electricity, and Volcanology:* Besides local changes in the distribution of the earth's magnetic forces in the neighborhood of volcanic eruptions, there are other points of joint interest to students of volcanology and of terrestrial magnetism and electricity. As an illustration may be cited the remarkable circumstance of a world-wide magnetic disturbance which took place almost simultaneously over the entire earth, beginning at 7 h. 54 m. St. Pierre local mean time on May 8, 1902. According to the reports at the time of the Mont Pelée eruption the town clock at St. Pierre was found stopped at 7 h. 50 m.; how accurately this clock kept local mean time is, of course, not known and it may be that the time as obtained from the magnetic records will have to be regarded as the most accurate determination of the beginning of that remarkable eruption. Magnetic disturbances occurred again during the eruptions of May 20 and May 21, 1902.

At the meeting of the "Congress of Arts and Science," held at St. Louis, 1904, L. A. Bauer gave a preliminary discussion of the magnetic disturbances just referred to in connection with his paper on "The Present Problems of Terrestrial Magnetism."[8] The precise causal connection, if any, between the Mont Pelée eruption and the magnetic disturbance has not been completely worked out at present. The connection may have to be sought through the fact that there were remarkable electric phenomena during the eruption which may have caused a sufficient change in the electrification, or in the electric currents, of the earth's atmosphere to have produced in turn a magnetic effect.

During the eruption of Krakatoa, Java, of 1883, there was no world-wide magnetic disturbance. However, a local disturbance, of discontinuous character, occurred at the near-by Batavia Magnetic Observatory, this disturbance lasting merely during the rain of volcanic ashes upon Batavia, and the magnetic effect was attributed by the director of the Batavia Observatory to the magnetic character of the ashes. The electric phenomena in connection with the Krakatoa eruption were also very marked and changes of interest to students in terrestrial electricity may have occurred.

The question of electric currents in the earth's crust in connection with volcanic eruptions is also a matter of interest. Thus Palmieri[9] made earth-current observations in the vicinity of Vesuvius during the period 1889–1893. His observations seemed to show a parallelism with the

activity of the volcano. He furthermore found that the action of Vesuvius was such as to mask the diurnal variation in the observations of earth currents.

Reference may also be made to the investigations by Folgheraiter and others in Italy, B. Brunhes and P. David, in France, with regard to the magnetization of lava beds.

10. *Laboratory Problems in Terrestrial Magnetism and Electricity:* One of the prime purposes of the Section of Terrestrial Magnetism and Electricity should be to stimulate research in these fields as widely as possible so as to increase the number of workers, especially at our universities. About the middle of the last century very much of the investigational work in the United States in the subjects here considered was carried out by university professors. It must be one of our endeavors therefore to present and suggest problems which might be advantageously taken up at universities, either in connection with laboratory work or field investigations, over a region of greater or less extent.

Among some of the problems for experimental and theoretical investigation may be mentioned those with regard to the origin of the earth's magnetic field, origin and maintenance of the earth's electric field, improvement of instrumental appliances for magnetic and electrical measurements on the surface of the earth and at altitudes above, the effect of pressure upon the critical temperature of magnetization, studies in magnetism, in general, etc.

In conclusion, let it be stated that the incompleteness of this sketch of some of the chief problems of concern and interest to the Section of Terrestrial Magnetism and Electricity is fully realized. Other topics of importance could profitably have been dilated upon, especially with regard to the question of the origin of the earth's magnetic field and as regards certain outstanding problems in the fruitful subjects of atmospheric electricity, earth currents, polar lights, upper-air electric phenomena bearing upon radio-telegraphy, etc. It must suffice, owing to the limits imposed upon the present paper, merely to have called attention to these additional problems, and to reserve fuller treatment for a subsequent occasion. It is hoped, however, that enough has been said to show the intimate relation of the various problems not only to the sister branches of geophysics represented by the other sections of the American Geophysical Union, but also to the general subjects of physics, astronomy and geology.

[1] See *Terrestrial Magnetism and Atmospheric Electricity*, **9**, 1904 (127, 128).

[2] *Philosophical Magazine* for December, 1919 (669–684).

[3] *Proceedings Royal Society, A*, **95**, 1918 (61–83); also *Monthly Notices of R. A. S.*, Nov., 1918 (70–83); and *Observatory, London*, **42**, No. 539, May, 1919 (196–206).

[4] See L. A. Bauer's "Resumé of Observations Concerning the Solar Eclipse of May 29, 1919, and the Einstein Effect," *Science*, March 26, 1920 (201–311).

[5] See article by S. J. Mauchly and A. Thomson, *Terrestrial Magnetism and Atmospheric Electricity* for June, 1920.

[6] *Terrestrial Magnetism and Atmospheric Electricity*, **24**, June, 1919 (96).

[7] See *Terrestrial Magnetism and Atmospheric Electricity*, **19**, 1914 (57–72, 189–203); also *Bulletin of the Seismological Society of America*, **4**, 1914 (204–214).

[8] See *Publications of St. Louis Congress of Arts and Science*, **4**, 1904 (750–756).

[9] Palmieri, L., "Osservazioni delle correnti telluriche," *Rend. d'Acad. Napoli*, **3**, 1890 (225, 250); **4** (164, 228); **5** (216).

THE PROBLEMS AND FUNCTIONS OF THE SECTION OF PHYSICAL OCEANOGRAPHY OF THE AMERICAN GEOPHYSICAL UNION

By G. W. Littlehales

The former and present function of the ocean in the history of the earth and in its economy has forged bonds of kinship between oceanography and many other branches of science. Ever since the ocean became the world-encompassing highway of communication, its surface aspects, embracing the movements of the waters in waves, tides, and currents, have been subjects of observation. With the advance of the physical sciences and a knowledge of the extent of the ocean came the realization that so large an expanse of a substance having the highest known capacity for heat must, to a large extent, govern the external temperature of the earth and exercise an important influence as a factor in geophysics. But centuries of voyaging did not extend marine observations beyond the delineation of coasts and the service of navigation; and, in the middle of the nineteenth century, the sea remained unfathomed, and the observations of the physicist, the chemist, the geologist, and the biologist did not extend beyond the shallow coastal waters.

In setting forth the principal deep-sea expeditions, by nations and states, through the names of the vessels engaged and the period of their service, we shall serve ourselves the purpose of reflecting the progress of the attempts that have been made to ascertain the physical characteristics of that vast region of the earth's surface which is occupied by the deeper waters of the ocean:

Austria
Pola (1891–1910).
Belgium
Belgica (1897–9).
Denmark
Ingolf (1895–6).
France
Travailleur, Talisman (1880–3).
Caudan (1895).
Français (1903–5).
Germany
National (1889).
Valdivia (1898–9).
Gauss (1901–3).
Planet (1906–14).
Great Britain
Lightning (1868).
Porcupine (1869–70).
Challenger (1873–6).

Investigator (1887–1902).
Discovery (1901–4).
Scotia (1902–4).

Holland
William Barents (1878–84).
Siboga (1899–1900).

Italy
Washington (1881–2).
Vettor Pisani (1882–5).

Norway
Voringen (1876–8).
Fram (1893–6).
Gjoa (1903–5).
Michael Gars (1900-20)

Russia
Vitiaz (1886–9).

Sweden
Vega (1878–80).
Antarctic (1901–3).

United States
Albatross (1883–1920).
Blake (1876–97).
Narragansett (1871–3).
Nero (1900).
Thetis (1895).
Tuscarora (1873–6).

Principality of Monaco
l'Hirondelle (1885–8).
Princesse Alice I (1891–7).
Princesse Alice II (1895–1914).

These expeditions and many others of lesser import, operating for the most part in seas remote from the countries in which they were fitted out, have contributed much of the literature of oceanography in which we find set forth the dynamic meteorology and climatology of the ocean, the theories of the tides and waves and the observed facts concerning them, the depths of the ocean, the temperature, the composition and circulation of oceanic waters, the nature and distribution of marine organisms at the surface and in the depths, and the origin and distribution of marine deposits over the floor of the ocean. But the ocean is so vast that the accumulation of facts of observation concerning it—extensive though it be—is but a sparse array in geographical distribution and constitutes but a skeleton of knowledge in relation to the configuration of its basins, the nature and distribution and thickness and stratification of the deposits which cover the bottom, and the physical and chemical properties and movements and mode of operation of its waters in producing their effects in the economy of the earth.

It is not alone through expeditions upon the ocean that oceanography has progressed; investigations in marine laboratories and institutions of research and discoveries in cognate sciences have sometimes yielded more advancement than distant and perilous voyages.

Advancement in the nature of the application of the philosophy of method has enabled oceanography to profit in its later stages of development. The system according to which progress is now being sought is the study in detail of definite stations in the ocean occupied in concert, and, as we hope it will be, by international coöperation, and periodically revisited for the purpose of observing the variations of physical condition whose import, when it comes to be understood, will enhance all those wealth-producing sources which operate in seasonal cycles. Observa-

tions of temperature, salinity, gas content, and currents made as nearly as possible at the same instant at series of points or stations and throughout a network of lines distributed in the depths beneath a given area of the ocean, and repeated every three months have afforded the means of making synoptic charts which disclose the existence of bends or undulations like the waves formed on the boundary surface between water layers of different densities. It is the mathematical investigation of the variations with time of the changing network of lines of equal values of the physical elements in their distribution in the depths that promises to introduce oceanography into the ranks of the exact sciences by enabling oceanographers, by mathematical laws, to predict effects from a few observations strategically placed.

Conspicuous among the features of the resumption of American oceanographical operations, after the interruption occasioned by the exigencies of the late times, are the following: The International Ice Observation and Ice Patrol Service in the North Atlantic Ocean, employing the vessels of the United States Coast Guard under an arrangement by which the cost is shared proportionately by the nations participating in the London Conference of 1913, is engaged (coördinately with the primary duties of ascertaining the locations and progressive movements of the limiting lines of the regions in which icebergs and field ice exist in the vicinity of the Grand Bank of Newfoundland and the dissemination of the information so ascertained for the guidance and warning of navigators) in gathering an important accumulation of oceanographical and meteorological observations. Year by year, observations at recorded times, extending from the surface to the bottom, are made in well determined geographical positions throughout the patrolled region for determining the temperature and salinity of the water by readings in series at definite depths, the direction and rate of movement of the waters in the different depths, the collection and preservation of plankton and samples of the water from ascertained depths, and in recording the state of the weather and the sea together with the barometric pressure, the humidity and the temperature of the air. These observations are published annually in the Bulletins of the United States Coast Guard, Treasury Department.

Closely related to these investigations from the standpoint of the advancement of oceanography, is the accumulation of observations resulting from the annual returns of the schooner Grampus in the Gulf of Maine and its vicinity, for the study of the correlation between physical oceanography and biological oceanography in these waters, under the joint auspices of the United States Bureau of Fisheries and the Museum of Comparative Zoölogy of Harvard University.

At La Jolla, near San Diego, California, there has grown up an institution by the name of the Scripps Institution for Biological Research, whose operations, recently brought under the auspices of the University

of California, constitute an exemplar of intensive oceanographical investigation. By systematically and repeatedly tabulating and mapping standardized values of the temperature, salinity, density, currents, and gas content of the water of the Pacific Ocean, serially observed at ascertained intervals of depth from the surface to the bottom in fixed locations, the variations of these physical elements, with time and locality, in their distribution in the depths, have been revealed to an important extent within the confines of the oceanic tract in the region of the seat of the Institution, stretching from San Diego to Point Concepcion and embracing an area of more than 10,000 square miles.

It is the present purpose of the Section of Physical Oceanography to foster the labors of these agencies and the similar ones which are contributed by the Navy and the Coast Survey and to seek opportunities to supplement them and link their operations, as far as may be, into coördination with the operations of the oceanographers of Japan, of Australia and New Zealand, of the North Sea International Council of Exploration, and the Mediterranean Sea International Council of Exploration. And, through the formation of committees, to provide that consideration shall be given to the problems of evaporation and heat transference and the interrelations between oceanography and meteorology, to the problems of dynamic oceanography including the variations of mean sea-level and the tides and their manifestations in the depths as well as the surface, to the investigation of the chemical and physical properties of the waters including the penetration of light, to the investigation of the origin and distribution of bottom deposits, to the problem of ascertaining the conformation and topography of the basins, and to the ways and means of advancement in the domain of physical oceanography.

THE PROBLEMS OF VOLCANOLOGY

BY HENRY S. WASHINGTON

INTRODUCTION

Of the various sciences represented in the American Geophysical Union that of volcanology is perhaps the most complex and has probably most points of contact with the other geophysical sciences. This complexity and variety in the problems presented by the study of volcanoes arises, in part, from the fact that they are, as has been well said, "natural laboratories." Also the distribution and many of the activities of volcanoes are closely connected with some of the physical, as well as the chemical forces that are involved in the formation and in the present condition of the earth.

In presenting some of the main problems of volcanology, we may begin with those that are essentially and more purely volcanological, and

then take up seriatim the consideration of certain problems in which the other sciences represented by sections of the Union may enter. Because of the limitation of space only a brief enumeration and presentation will be made, and no adequate discussion will be undertaken. I am much indebted to various members of the Union for some valuable suggestions, which will be mentioned in their proper places.

VOLCANOLOGY

A word may be said as to the name of the science. The spelling, *volcanology*, (rather than *vulcanology*), has been adopted officially by the Executive Committee of the Union, following the report of a sub-committee. The decision was arrived at chiefly in accordance with the dictum of the Century Dictionary (*s. v. vulcanism*): "The words *volcano* and *volcanic* are firmly fixed in English, and the former is in universal and exclusive use among those who speak that language. Hence all the derivatives should be spelled accordingly."

It is now well recognized that there are different types of volcanoes, distinguished by their form and structure, as well as (generally) by the differing kinds of volcanic activity, and that these differences are to a large degree dependent on, or at least coincident with, the differing chemical types of the volcanic material, such as, for instance, whether rhyolitic, andesitic, or basaltic. It follows that the activity or form of any one volcano, such as Vesuvius, Kilauea, or Stromboli, cannot be taken as typical of all volcanoes generally. This important principle does not seem to be generally recognized.

One of the first and most fundamental needs for the proper study of volcanoes is the compilation of a complete and fairly detailed catalogue of volcanoes (suggested by Wood and the writer). Such a catalogue should list all known active, dormant, or recently extinct volcanoes. It should give briefly, so far as known or ascertainable, the geographical position; the hypsometrical and areal data, including those of the crater; the general petrographical characters of the lavas, with the general order of succession of the different types; whether active, dormant, or extinct, with the geological period of commencement if possible; the general types of volcanic activity; a chronological list of the known eruptions, with some indication of the degrees of intensity; and a bibliography of the chief sources of information as to each volcano; with possibly such other data as may present themselves.

The older lists, as those of Daubény (1848), Landgrebe (1855), Scrope (1862), and Fuchs (1865), are far from being complete; while even the more recent ones, such as that of Mercalli (1907), though they list many volcanoes not known to the earlier volcanologists, are yet themselves not complete, nor do they give many of the varied data mentioned above as desirable. The compilation of such a catalogue might be entrusted to a special committee.

Another great desideratum is the keeping of a systematic record of all known volcanic events, eruptions, indications of renewed activity, and so on. This should be as complete as possible, and might be compiled by another, or the same, special committee, possibly with the assistance of some news-clipping bureau and correspondents at various volcanoes. In such a record, and in all other official volcanological literature, the sensational features should have a minimum of stress laid upon them.

If possible, some systematic investigation of the little known volcanoes and volcanic regions should be undertaken. This applies especially to those of the islands in the Pacific and Atlantic Oceans, and Daly, in an important paper, has called attention to the paucity of our knowledge as to the first of these. It is possible that such an investigation may be started by the coming Pan-Pacific Congress at Honolulu, at which the scheme will probably be proposed. For this purpose the assistance and coöperation of intelligent persons living in, or trading among, the various islands may be obtained, as well as, possibly, that of some private expeditions. Even the collection of chance specimens from the little-known islands would be of great value.

In this connection it may be suggested that the early history of our own western volcanoes should be investigated, as by search through the records of the early explorers and settlers, for possible mention of volcanic activity, and the verification or disproof of some such reported occurrences. The lavas of these volcanoes are well known, but the character of their volcanic activity has been much neglected.

Apart from the purely scientific question as to how and why a volcano "works," is the practical, and scientifically equally interesting, one of the prediction of eruptions. Unfortunately, with two or three exceptions, we know of volcanic activity almost entirely as displayed only during eruptions, and especially major ones. It is much like studying a complicated machine only when it is working at full speed; we can understand better the relations of the parts and their working if the machine is examined when the parts are moving slowly or are at rest.

For the prediction of eruptions, and for the solution of many other volcanological problems, prolonged, continuous, and systematic observations on volcanoes are absolutely necessary, not only during eruptions, but during the quiet intervals that precede and follow eruptions. Such observations have been carried on at Vesuvius, at first by Mercalli from 1892 to 1906, and by Malladra from about 1912 to the present time. Similar observations have also been carried on for nearly ten years at Kilauea by Jaggar, at times with the assistance of Perret, and Wood. It is hoped that the very detailed observations of Malladra will soon be published; they will form an invaluable contribution to volcanology. The records of the Kilauea observing, kept by Jaggar since 1911, are especially full and detailed and constitute what would be, if published, one of the

most valuable contributions to volcanology yet made. It is of the greatest importance to our science that both these stations be maintained, and their observations published, so as to continue the important work so well begun. And it is also important that similar stations be established and maintained at other volcanoes of types different from these two. In its type of activity Kilauea is very unusual, while Vesuvius, though more normal in its form of activity, is very unusual in the character of its ejected material. Stations for continuous observations over a long series of years are needed at volcanoes of the rhyolitic, andesitic, and basaltic (besides Kilauea) types. These should be established at volcanoes which are more or less continuously active, and with a fair prospect of some eruptions from time to time. Several volcanoes in Japan and Java suggest themselves as favorable, and there might be mentioned also Apia in Samoa, Izalco in San Salvador, Stromboli and Santorini in the Mediterranean, and some volcanoes along the Andes and in Mexico. The establishment of such stations offers, of course, many difficulties, but it would be well for the Section to bear this in mind as one of the most important, even if one of the most difficultly realizable, objects of effort.

At any such station the continuous observations to be made would be many and various, including, for instance, daily notes on the state of the crater and fumaroles, the noises emitted, the microseismic vibrations, the weather and barometric data, etc. With all this, there would be opportunity for the collection of specimens from the various flows, fumarolic salts, gases from the lava; measurement of the earlier flows; observation of the temperature of the lavas; and many other matters. The collection of volcanic gases, and study of the best methods for this, are of special importance.

It is not probable that many of these stations will be maintained, at least in the near future, and, as the study of the phenomena of great eruptions is also of great importance, it would be very desirable if some arrangements could be made (possibly international as well as national), through which any usually dormant volcano that was reported to be showing signs of renewed activity or to be in a state of eruption might be visited immediately by competent observers. This has been done for several great eruptions, such as Santorini in 1866, Mont Pelée in 1902, Vesuvius in 1906, and Sakurajima in 1914; but always in a haphazard way, thus diminishing materially the value and scope of the observations in some cases. The immense value of such visits, however, is shown by such monumental works as those of Fouqué on Santorini, the British and Dutch at Krakatoa, and of Lacroix at Pelée, of Perret, Omori, and Kato at Sakurajima, with others of scarcely less importance. It may be mentioned here that we may hope soon to have a similarly complete and monumental report by Perret on the Vesuvius eruption of 1906.

Among other problems with which volcanology has primarily to deal the following may be mentioned:

Study of the rôle played by earth strains is suggested by Wood, who points out that, although such factors as accumulated gas tension and possibly others, may enter, yet "a general cause of volcanic extrusion [may be] due to the so-called mountain building forces as builders of strain, and to certain variable stresses acting as trigger forces." He suggests that study of this phase of volcanology would be of interest also "to the geological seismologist, the physical geologist, and the student of isostasy." The importance of arriving at some estimate, however crude, of the magnitude of volcanic forces, possibly as a measure of crustal deformation forces, is pointed out by Wright.

A systematic and properly conducted study of thermal gradients in the crust is suggested by several (Daly, Van Orstrand, and the writer). Such a study should be carried out at carefully selected points, the stations being chosen with due regard for both the proximity and the absence of volcanoes, differences in the underlying rocks and their geological structure, the absence of secondary sources of heat (such as the proximity of ore bodies), and with proper and well-controlled means of measuring the depths and temperatures. It is reasonable to suppose that such a systematic project might be carried out with no very great difficulty or prohibitive expense. It is needless to say that the results obtained by such properly controlled means would be of the greatest value for our study of the interior of the earth, and would supersede many of the, for the most part, haphazard and not mutually comparable data that are now available.

Connected with this matter is that of the investigation of the thermal gradient and the sub-surface conditions in the mass of an active volcano, both on its flanks and within the crater. Such investigations might throw light on the rôle of gas inter-reactions as causal in the production or maintenance of volcanic heat, as has been suggested by Day, and they might also be of great practical importance in the study of the possible utilization of volcanic heat as a source of energy, a matter to which my attention was directed in Italy, both before the war and during the past year. The study of the possibilities of regions of geysers and hot springs, as the Yellowstone, and in the Hawaiian Islands, along this line, is also suggested. In this connection the present successful utilization of the soffioni at Lârderello in Tuscany by Ginori Conti may be recalled.

The problem of radioactivity as a source of heat, and the possible formation of a special committee for this purpose, is suggested by Daly, who also suggests the study of the "meaning of the earth's rigidity."

As minor matters may be mentioned the study of geysers and their mode of action, and also the formation or unification of a terminology for

lava structures, since "aa" and "pahoehoe," the two terms most commonly used, by no means cover all the types that are met with.

VOLCANOLOGY AND GEODESY

The apparently intimate correlation between the average chemical composition of the igneous rocks of the continents, oceanic floors, and of smaller petrographical provinces (as a means of arriving at comparable densities of their masses), and their elevations, together with the bearing of this study on the theory of isostasy, has been pointed out recently by the writer. W. Bowie suggests the continuation of such investigations "as a contribution to the study of isostasy." He also suggests the advisability of establishing gravity stations in the immediate vicinity of volcanoes, the selection of appropriate localities to be made with the coöperation of volcanologists. He thinks that there is reason to believe that "areas where volcanoes are active are probably in a high state of isostatic equilibrium," and the determination of this and analogous points would be of great service to both sciences.

VOLCANOLOGY AND SEISMOLOGY

At the outset it must be emphasized that, contrary to a wide-spread belief, there is little connection between volcanoes and most earthquakes, though their frequently contiguous localization may be due to one and the same cause, such as a fracturing of the earth's crust. By far the great majority of recorded earthquakes (it has been estimated at about 95%), and these the most important, are due to crustal movements and, so far as we know, have no direct connection with volcanoes or their activity. The small remainder of earthquakes of volcanic origin are mostly very local and of comparatively small magnitude. Notwithstanding this, the two sciences are in contact at several points.

The study of the small number of truly volcanic earthquakes is, of course, intimately connected with the study of the volcanoes that produce them; while, conversely, the question of the influence of earthquakes as possibly initiating volcanic eruptions or other activity is an interesting field for study.

The study of volcanic tremors of small amplitude and energy, during periods both of quiet and of eruption, is of great importance, and would naturally form part of the systematic, continuous study carried on at a volcano station, as has been done both at Kilauea and Vesuvius. This may be expected to be of very material aid in the future prediction of eruptions. For such observations some of the forms of seismographs and other seismological instruments, duly modified to suit the different conditions, are necessary, and seismological principles or data would also enter into the study.

VOLCANOLOGY AND METEOROLOGY

As has been already noted, the systematic observation of certain meteorological data, especially barometric pressures and rainfall, would form an

important item in the routine observations at a volcano station; the former entering into the question of the "trigger forces" needed to start an eruption, and the latter as bearing on the question of the origin of the water present in volcano clouds and gases.

Humphreys called attention some years ago to the influence of the finest volcanic dust, blown into the upper regions of the atmosphere during first magnitude eruptions and suspended there for prolonged periods, in lowering the solar constant of radiation, and thus affecting the weather over wide areas for long periods. This has been invoked to account for periods of glaciation. A knowledge of the chronology of volcanic eruptions is necessary for their proper correlation with meteorological and radiation data, and the further study of this theory.

In this connection mention may be made of the possible utility of the study and identification of volcanic dust collected at great distances from a volcano in eruption, as bearing on the study of the currents of the upper atmosphere.

The possible influence of the supposed blanketing effect of carbon dioxide on past climates has long been a favorite hypothesis, and as it is known that this gas is given off by volcanoes, a possible line of research is indicated connecting climates with periods of great volcanic activity—a matter that has already received some attention.

A minor problem, and one that has been little studied, because of the rarity of its occurrence, is that of the blue or green color presented by the sun during some eruptions, as that of Krakatoa in 1883. Especial attention might be paid to this by the observers of future eruptions.

VOLCANOLOGY AND TERRESTRIAL MAGNETISM

Bauer calls attention to the "local changes in the distribution of the earth's magnetic forces in the neighborhood of volcanic eruptions." He also lays special stress on the investigation of such phenomena as the world-wide magnetic disturbance that took place coincident with the eruption of Pelée in May, 1902, and the fact that no such world-wide disturbances seem to have occurred in connection with that of Krakatoa in 1883. This is a problem in which the observations of local observers might be of great value, were their attention called to its importance and their observations properly directed. The lightning flashes in the volcanic cloud, and other such phenomena, during eruptions, are also of interest and are probably worthy of study.

Bauer also suggests the problem of electric currents in the earth's crust during volcanic eruptions, such as have been observed by Palmieri at Vesuvius. A minor point is the magnetization of lava flows, and that of clay beds covered by them; while there might also be mentioned here the study of fulgurites—the vitreous tubes and patches formed by lightning strokes fusing the rock of the summits of volcanoes, (as Ararat), and other places.

VOLCANOLOGY AND PHYSICAL OCEANOGRAPHY

The most important point of contact between these two sciences is presented by submarine eruptions. Far too little is known of these occurrences, either as to their characters or their distribution over the ocean floors. For their study a most important and but little gleaned field of information is furnished by ships' logs, and I would suggest here the advisability of examining the thousands of ships' logs for records of submarine eruptions. This might be done through a duly appointed committee, with the coöperation of the Section of Oceanography, and with the assistance of the naval and other maritime authorities and ship owners, in whose hands the logs may now be. The data revealed by such an investigation would be invaluable for the study of submarine eruptions, and also for that of other important geophysical problems, such as earth fracture lines and lineaments, the origin of "tidal waves," etc. The submarine cable companies might also furnish valuable data on this subject from their records of broken cables, as in the case of the Aeolian Islands. I recommend this project most strongly for the consideration of the two Sections most interested.

Another problem is that of the material of the deep sea deposits, the study of which was first taken up by the "Challenger" expedition. The "red clays," as is well known, are composed largely of the decomposition products of volcanic material, and their study should be carried out more systematically and extensively than is being done at present. Further study also of the included, and too little known, chondritic and manganese nodules would also be of interest; while the study of blocks of rock (many of them volcanic) dredged from the depths also presents many features of interest. Some provision or arrangement might be made for the preservation and study by some central body of any such material that may be collected in the future. In connection with this the importance is urged of making special effort to obtain rock specimens, or sea floor deposits, from the immediate vicinity of submarine eruptions, or from areas of the sea floor that are known to be specially subject to such disturbances. These would throw much light on the petrographical and other characters of such volcanoes, and the information thus yielded would be very useful in several ways.

VOLCANOLOGY AND GEOPHYSICAL-CHEMISTRY

It is needless to dwell on the close connection between these two sciences —closer indeed than that between any other pair of the geophysical sciences. Some phases have already been alluded to on a previous page of this report, and the matter is also touched on in the portion devoted to the Section of Geophysical-chemistry. It will suffice here merely to indicate briefly some of the main points of interest.

The study of the formation of volcanic rocks and of their component

minerals is one of the utmost importance to some phases of the study of volcanoes. Of similar importance is the study of the gases contained in volcanic and other igneous rocks, as well as of the gases given off by volcanoes in activity, with experimental laboratory study of their equilibria and possible interreactions. As is well known, all these, and similar problems form objects of investigations carried out at the Carnegie Geophysical Laboratory.

The formation of volcanic salts and other fumarole products, as well as the action of the fumaroles themselves, their temperatures and other characters, also come under the domain of geophysical-chemistry. Among other subjects may be mentioned: the determination of the actual temperatures of lavas of different compositions, and the thermal and luminous radiations of molten lava; the study of volcanic flames with the spectroscope; the melting-points or intervals of solidified lavas, especially as compared with the temperatures at which they issue; the expansion of the included gases as a factor in producing the various types of lava texture, such as the scorias and pumices; the formation of jointing through cooling, whether on a large or a small scale; the specific heats, compressibilities, thermal expansion coefficients, densities, and other physical or physico-chemical data, of rocks and rock minerals; the viscosity of lavas and its relation to temperature, chemical composition, and gas content. Many others suggest themselves, but the possible list is too long for enumeration here.

VOLCANOLOGY AND MISCELLANEOUS SCIENCES

In a previous section we have already touched on the connection with engineering in the possible utilization of volcanic heat. There may also be mentioned the application of engineering to the prevention of destruction by lava flows or eruption-bred torrents of mud on the flanks of volcanoes.

In connection with botany and related sciences there may be mentioned the subject of the effect of volcanic gases on different kinds of vegetation, a study that is being made at Vesuvius. The problems of the reclamation for agriculture of areas covered with recent lava flows or thick deposits of volcanic ashes or dust is also one of great practical importance in many volcanic regions. It may be mentioned, in this connection, that the recent expedition to Katmai, under Dr. Griggs, was undertaken primarily for the study of the vegetation of the region devastated by the eruption. Though the conditions are almost diametrically opposed, the main object was like that of the study of the Salton Sea flora by the Carnegie Desert Laboratory. Study of methods for the commercial extraction of potash from certain types of lava, as those of the Italian volcanoes, is also a matter of possibly great importance for agriculture in the future.

Finally, to leave the earth, the suggestion is made that some knowl-

edge of the character and chemical composition of the rocks of the moon, and thus possibly a decision of the question of the volcanic or non-volcanic origin of its "craters," might conceivably be arrived at by astronomers, through study of the maximum polarization angle of various portions of the moon's surface, such as the floors of the "seas" and the "walled plains." From the determination of this, calculation of the index of refraction of the material is a simple matter. The difference between the refractive index of a rhyolitic glass (*ca.* 1.490) and that of a basaltic glass (*ca.* 1.590) is so great that it would seem to be possible to obtain some general or at least roughly approximate knowledge of the general composition of the surface rocks of the moon, especially if they are glassy lavas. We should be able, in any case, to determine thus whether all parts of the moon's sur face are composed of the same material.

AN OUTLINE OF GEOPHYSICAL-CHEMICAL PROBLEMS

By Robert B. Sosman

The subject-matter of geophysical-chemistry may be defined as "the physical properties and chemical reactions of the substances and aggregates that make up the earth". It may therefore be roughly divided into two parts: A. Properties and reactions of materials accessible at the earth's surface. B. Properties and reactions of materials in the earth's interior.

Each of these may again be sub-divided as follows:

1. Properties and reactions of individual chemical substances; for example, the silicate minerals.

2. Properties and reactions of aggregates; for example, oceanic water, silicate rocks.

3. Properties and reactions of larger units of matter; for example, glaciers, batholiths.

A. MATERIALS AT THE EARTH'S SURFACE

Chemical Substances

A relatively small number of "common" oxides serves to make up practically 98% by weight of the outer ten miles of the lithosphere. All the other elements and compounds known to chemistry are included in the remaining 2%. From the geochemical standpoint, therefore, we may divide chemical substances into two classes: "abundant" and "rare."

The "abundant" oxides are, in the order given by averages of a great many analyses of terrestrial rocks, as follows:

SiO_2	About 60 per cent by weight
Al_2O_3	About 15 per cent by weight
FeO, Fe_2O_3	About 6 per cent by weight

CaO	About 4.9 per cent by weight
MgO	About 3.7 per cent by weight
Na_2O	About 3.3 per cent by weight
K_2O	About 3.0 per cent by weight
H_2O	About 2.0 per cent by weight
CO_2	About 0.7 per cent by weight

An understanding of the chemistry of these oxides and their combinations is essential to the progress of petrology. Their study should proceed from the simple to the complex, i.e., should begin with the individual oxides, then proceed to their two-component systems, then the ternary systems, and so on. Upon this fundamental basis is then erected the structure of physical properties for each system: densities at all accessible temperatures, mechanical properties, fluidity, surface tension, specific and latent heats, etc.

The study of these systems may be divided on practical grounds into (1) investigations of the anhydrous oxides and silicates (taking in the first eight oxides in the list above); (2) investigations involving *hydrous* silicates, as well as combinations containing both carbon dioxide and water.

(1) *Anhydrous Silicates.*—Work on the first group involves high-temperature researches under ordinary atmospheric pressure conditions, except in the case of systems containing the oxides of iron, where the oxygen pressure must be controlled, and systems containing the alkali silicates, where attention to moisture, carbon dioxide, and volatility of the oxides is necessary in certain cases. Considerable progress has been made in the study of the anhydrous silicates. The phase rule diagrams of the four ternary systems of SiO_2, Al_2O_3, MgO, and CaO are now complete, and a large amount of data is at hand on the alkali feldspars, the forms of silica, portions of several quaternary silicate systems, etc.

(2) *Silicates with Volatile Components.*—Work on systems involving the volatile components CO_2 and H_2O must be done, for the most part, under pressure, and with apparatus designed especially for this purpose. The methods are well in hand and progress is being made in assembling experimental data. The theoretical side, involving the complications due to pressure as a variable in addition to temperature, is also being carried forward by several investigators.

So much for the 98%. But the remaining 2% contains many natural substances of such great economic as well as geologic interest that they must also receive attention. These may be roughly classified as in the following examples:

The Sulfide Ores (e.g., sulfides of iron, nickel, zinc, copper, lead, cobalt, cadmium, mercury, silver).—These must be studied both in their dry melts (to obtain their fundamental characteristics) and in relation to water solutions under atmospheric pressure (problems of oxidation and

secondary enrichment). A distinct problem of the sulfides is their relation to the silicates in the igneous rocks (differentiation of sulfide-bearing bodies, as at Sudbury, Ontario).

Volcanic Gases and Salts.—These are of particular interest in their relation to volcanic activity, as at Kilauea and Vesuvius. Research on gases, including the various gas mixtures evolved from volcanic vents, is of a peculiarly trying character on account of the invisibility and intangibility of the substances handled, as well as the difficulty of collecting and transporting samples of the natural products. A special phase of this work is the study of the complex gases given off by fumaroles and hot springs. In addition to chemical composition and equilibria of the gases, data are needed on the physics of the flow of such gases from vents, as related to volume, temperature, and pressure at the point of emission.

The volcanic "sublimates", such as sulfur, ammonium chloride, arsenic sulfide, copper chloride, magnetite, may be mentioned in this connection, as well as the minerals accompanying fumarole and hot-spring activity.

The Oxide Ores (e.g., ores of iron, chromium, manganese, tin).—The study of these ores involves high-temperature investigations similar to those on the silicates, and also studies of the hydrated and colloidal oxides.

The Natural Hydrocarbons.—Organic chemistry of a very complex kind is involved in the formation and alteration of natural gas and petroleum, and many problems of physics and physical chemistry, such as adsorption, surface tension, and colloid phenomena, are also involved in their underground storage and movements.

Other substances—for example, the silicate ores, the carbonate ores, the titanium minerals—may be similarly grouped for purposes of experimental study, but it is hardly necessary here to make a complete inventory of such groups.

Running along with all these investigations is the general research necessary to develop experimental methods and apparatus, and to keep the general theory of physics and chemistry abreast of the newly accumulated facts.

AGGREGATES

In the preceding paragraphs we have mentioned some of the researches that are necessary on the chemical *substances* of the earth's surface. We come next to *aggregates*, including the igneous rocks, the pyroclastic and sedimentary rocks, the oceans and other bodies of water, and the atmosphere.

Igneous Rocks and Magmas.—Among the aggregates the igneous rocks are the most important, constituting as they do the original matter from which the others have been derived. They are poly-component systems with seldom fewer than six oxides, and it seems out of the question at present to give a complete phase-rule discussion of the chemistry of any

such complex system. It does seem possible, however, to cover the field of actually occurring rocks by partial systems based upon stable minerals, themselves combinations of two or more oxides.

Experimental work has progressed far enough to show that the fusion and solidification phenomena of the igneous rocks are capable of systematic treatment of the kind mentioned above, even when the volatile components H_2O and CO_2 are included.

Following this fundamental information on the fusion diagrams, we must know the densities of the igneous rocks and their magmas at all temperatures, with their changes of volume during solidification; the textures and structures produced by various conditions of solidification; the latent heats concerned in fusion of the magma and in assimilation by it of other rocks; and many other physico-chemical data.

Some of the phenomena of differentiation of silicate rocks are probably to be treated as results of simple crystallization backed up by the effects of gravity in causing sinking or flotation of crystals. This is the only method of differentiation that has been experimentally proven, but the separation of two or more liquid phases, and perhaps other phenomena also, may take part in this little-known process.

The phenomena of the movements of igneous magmas in the lithosphere must next be attacked. This involves studies of viscosity and its changes with temperature; energy transfers accompanying movement, including perhaps the conversion of potential gravitational energy into heat energy by movement; differentiation due to movement; and the effects of the separation of gaseous constituents in causing differentiation, movements of magmas, and transfers of energy.

While the igneous rock is solidifying, processes of metamorphism may go on as a result of the passage of its more volatile constituents through the igneous rock itself and through its bordering rocks. The rise of temperature in the inclosing rocks will also initiate reactions and movements of material. The chemistry of these processes (alteration, serpentinization, contact metamorphism and replacement, formation of pneumatolytic dikes and veins) will have been included under the chemistry of the silicates with volatile components, but the rates of the reactions and the transfers of energy accompanying them must be studied as a separate problem. The question of the permeability of crystalline substances to volatile components may perhaps arise. Studies of the inversion of polymorphic forms, properties of liquid inclusions, and the effects of increased temperature on neighboring rocks must be made in order to obtain data on the original temperature of the magmas.

Sedimentary Rocks.—No sooner does the solidified igneous rock find itself at or near the earth's surface than it becomes the object of the group of processes known as "weathering." This is a subject that has been nearly at a standstill since the publication of Merrill's "Rocks, rock

weathering, and soils" about fifteen years ago, which summarized our knowledge up to that date. It involves physical factors such as the disintegrating action of periodically fluctuating temperatures, together with the study of relative rates of reaction within and in the presence of dilute water solutions, with the complications due to colloidal phenomena and reaction in capillary spaces—a difficult field and one worthy of well-planned effort.

Rock disintegration, corrasion, and transportation, included in the general process of erosion, bring in physical and mechanical problems such as the rounding of fragments by attrition; their sorting by air and water movements; their distribution and redistribution through the agency of winds, currents, and waves; the production of particular structures such as ripple marks; the movement of unconsolidated sediments as in sand dunes, soil creep, and solifluxion; and similar questions. Reference need only be made here to summaries of these problems by Vaughan and by Merwin before the Geological Society of America in December, 1919. Experimental physics will be found deeply involved in all such problems. Colloidal chemistry will also enter, as, for instance, in the question of the precipitation and re-solution of fine suspensions; likewise biological chemistry, in the precipitation and consolidation of calcium carbonate, ferric oxide, and other products associated with organisms.

Chemical questions allied to those of weathering will enter into the problems of the consolidation and alteration of sedimentary rocks, including cementation and recrystallization, the formation of low-temperature veins, silicification, and the growth of concretions. The elastic constants of porous aggregates offer an example of the physical data that are likely to be needed in this same connection.

Pyroclastic Rocks.—The question of the origin of these rocks brings up the problem of explosive volcanism. In its larger aspects this belongs properly to the Section of Volcanology, but the products of this type of volcanic activity are very wide-spread, and the particular physical and chemical questions arising from the state of subdivision and modes of distribution and alteration of the products deserve special mention in this rather rough grouping of geophysical-chemical problems. Examples are: the molecular mechanism of the explosion, the peculiar properties of flowing dust-clouds, the physics of the projection of volcanic bombs, the sorting of products by air currents and the weathering of "ash" to form colloidal products.

The Metamorphic Rocks.—The point where a rock ceases to be "igneous" or "sedimentary" and becomes "metamorphic" is not now exactly defined, though subject to exact definition, but a distinct set of physical and chemical questions undoubtedly enters into the problems of dynamic and thermal metamorphism. The effects of *non-uniform* pressure in causing the flowing of crystalline substances and aggregates and their

solution and redeposition, as well as the forces exerted in their recrystallization, are of particular interest. The structures resulting from movement in silicate rocks under differential pressure are in need of quantitative study. Transfers of mechanical energy and its conversion into heat are also involved. Consolidation and recrystallization as a direct result of *uniform* pressure are likewise to be considered.

Mention should be made in this connection of the physics and chemistry involved in faulting as well as in jointing and minor movements of the solid rocks. These phenomena have often been treated under the principles of elastic theory as applied to homogeneous bodies, yet there can be no question that the elastic properties and conditions of rupture of aggregates must differ in many essential particulars from those of homogeneous bodies. Here is a considerable field for experimentation.

Bodies of Water and the "Chemical Sediments".—The chemistry of the deposition of salts from sea-water has already been made the subject of special research, and van't Hoff's results in this field are already familiar. The deposition of calcium carbonate awaits a similar thorough study. Allied questions are the formation of dolomite, the deposition of various salts from inclosed bodies of water, the deposition of phosphate rocks, the precipitation of colloidal suspensions of clay and other substances, and the origin of the great deposits of sedimentary iron ore.

Problems of fresh-water bodies and streams include the chemistry of bog iron ores, and the amounts of insoluble and colloidal solids and of soluble salts carried by streams. Problems relating to underground waters may also be included here, such as: the dissolving and recrystallizing activity of underground waters; connate waters and their possible chemical changes; movements of underground waters and their relation to the alteration and concentration of gas and oil; principles of hydraulics governing the flow of wells and springs; and the characters which will serve to differentiate between vadose and juvenile waters.

The Atmosphere.—The physics and chemistry of the atmosphere considered as a chemical unit or aggregate, while logically a part of geophysics and geochemistry, are usually considered to be part of the province of meteorology; and in fact most of the investigations in this field are being made by meteorologists, or physicists employed in meteorological organizations. These questions will therefore not be further considered here. Data bearing on the origin of the earth's atmosphere and its possible changes of composition in the past—for example, the composition of the gases found in rocks and the gases dissolved by the waters of the oceans—are, however, of direct interest to the present Section.

LARGER UNITS

The larger units of matter at the earth's surface are for the most part covered by other Sections. The atmosphere as a unit is treated by the

Section of Meteorology and the atmospheric-electric branch of the Section of Terrestrial Magnetism and Electricity; the oceans by the Section of Physical Oceanography; the large land masses by the Section of Geodesy; volcanoes as units by the Section of Volcanology.

The larger geologic units (for example, petrographic provinces) and the rocks, considered as geologic units, might be considered to fall within the province of geology rather than geophysics, and the same may be said of the earth's glaciers and ice-sheets.

There are certain problems connected with these larger units, however, that may be neglected by the Sections mentioned, not from any lack of appreciation of the importance of the problems, but solely by reason of lack of training and lack of acquaintance on the part of their personnel with the technique involved; just as some of the geophysical-chemical problems mentioned above may be, relatively, neglected by reason of the geophysical-chemist's lack of training in other branches. For example, the physics of the flow of a glacier, considered as a unit, is not likely to be adequately handled by those trained only in the methods of glacial field geology, and it very properly becomes a subject for research under the present Section. The question of the chemical composition of a particular stratum of the atmosphere and the chemical equilibrium obtaining therein, while of great importance to the meteorologist, might fail of adequate treatment by an organization numbering no chemist on its staff. The distribution of certain rock-forming oxides according to "petrographic provinces" may be of primary interest to the geologist, yet the physico-chemical basis for that distribution—the question whether it represents an "original heterogeneity" or a result of differentiation—is a problem for this Section. The flow of rock aggregates of varying composition, again a matter of geological interest through being concerned in the processes of mountain-building, is a problem for the geophysicist. These examples will serve to indicate that there may be many points in which the Section of Geophysical-chemistry may be of direct assistance, through researches of its own type, in the work of other Sections.

B. PROPERTIES OF MATERIALS IN THE INTERIOR OF THE EARTH

The late G. K. Gilbert has made apt reference[1] to the earth's interior in these words: "Once it contained the forges of blacksmith gods; or it was the birthplace of our race, or the home or prison of disembodied spirits * * * * Science now claims exclusive title but holds it chiefly for speculative purposes." Upon the Section of Geophysical-chemistry more than upon any other will fall the duty of maintaining the validity of the title while bringing the property into productive use.

The outstanding difference of condition to which substances in the earth's interior are subjected, as contrasted with substances at the surface, is the tremendous difference in hydrostatic pressure. Research on

the properties of substances under high pressure is thus of first importance in relation to the interior of the earth. While we cannot at present go experimentally much beyond 12,000 megabars, a pressure corresponding to about 45 km. (about 29 miles of depth), yet a really adequate knowledge of properties in this range would give us an insight into the conditions in the interior beside which our present knowledge is equivalent to almost total ignorance.

It is most important to know in this connection the compressibility of the substances concerned, at various temperatures, and in both the liquid and the crystalline state, with its dependent constants such as change of melting-point with pressure, and effect of pressure upon solubility. Other important data are: the existence of new polymorphic forms of substances; the effect of pressure upon rigidity and its related elastic moduli; the effect of pressure upon diathermancy, thermal conductivity, specific heat capacity, and magnetic susceptibility; and the effect of pressure in modifying equilibrium in homogeneous as well as heterogeneous systems.

The properties mentioned in the preceding paragraph are all properties of substances in equilibrium. The effect of pressure upon rates of reaction and rates of diffusion and crystallization are also of importance. If the planetesimal hypothesis of the origin of the earth, or some modification of it, is true, it is possible that the interior is very far from a state of chemical equilibrium, and that redistribution of matter and energy may be going forward actively, with what results—whether rising or falling average temperature, increasing or decreasing volume, etc.—it is impossible now to say.

The preceding considerations apply also to the larger units of structure of the interior—although what these units may be we can only guess—and also to the earth itself as a unit. Complex mathematical analysis is necessary in applying the data to the units of structure as well as to the whole of a body as large as the earth, where the force of gravitation is itself variable with depth. These applications can in many cases be best made by those familiar with the properties in question as measured in the laboratory. The course of earthquake waves, to take an example of interest to the Section of Seismology, is dependent both upon the elastic constants of the earth's materials, and the possible reflection and refraction of waves at the boundaries of internal structural features.

In this connection, the members of this Section could do a service to the other Sections by making clearer the real meaning of some of the physical concepts and physical constants involved in geophysical problems. For instance, much confusion has been caused by the fact that there is more than one kind of "rigidity". The geologist to whom the statement that "the earth has the rigidity of steel" is rather vague may take temporary comfort from the fact that the statement also needs much qualification and explanation to the physicist.[2]

General Geophysics.—The physics of the earth considered as a unit (the classical "geophysics") is for the most part either covered by other Sections of the Geophysical Union or is customarily considered as a part of geology. Certain phases of geophysics, however, are not thus assignable and may be mentioned here in order that all of the groups of problems of the science may receive attention in this assembly of surveys.

The form and gravitation relations of our suspended spheroid, its magnetic and electrical properties, its properties as a vibrating body, and the physics of its air and water envelopes, are the obvious fields of appropriate Sections. Hypotheses of its origin and the logical deductions therefrom may confidently be left to the geologists, among whose faults that of narrow-mindedness and lack of a broad outlook in time and space have seldom been numbered.[3] Its properties as an absorbing and reflecting body for external radiation are being well handled by the astrophysicists and meteorologists.

The problem of the earth as a body radiating its own heat into space, however, is an example of a problem that may fail of treatment as a unit problem by any one of the above-mentioned groups acting alone. We are dealing not with a solid homogeneous spheroid of uniform surface temperature radiating freely into space, but with a rather heterogeneous body blanketed with several kinds of heat insulators whose composition varies both with depth and with time. Factors in the problem are: the production of heat by shrinkage; the contributions of heat from radioactive sources; the earth's present thermal conductivity and thermal gradient; the effect of varying carbon dioxide, water, ozone, inorganic dust, and clouds, upon the heat loss; and the effects of the possibly very different atmospheres with which the earth has been blanketed in past ages. The temperature at a given time and at a given distance from the center, as for instance at the solid surface of the land, depends upon a complex set of factors, and may well have been periodic in its variations.

The earth's volume and shape may have been similarly variable. In addition to the variation of temperature, already mentioned, the following are among the factors to be considered: the tides in the solid earth (on which extensive experimental work has recently been in progress) and the earth's properties as an elastic body; the viscosity of the earth as a whole, with relation to long-continued forces, and the existing state and method of maintenance of isostatic equilibrium in its surface layer; its breaking strength and form of rupture under forces changing too rapidly to cause flow; and the lag of elastic and viscous responses to changing forces, as in the case of the addition or removal of continental ice sheets.

Limitations of space forbid more than a sketchy outline of the problems set before the Section of Geophysical-chemistry, but it is hoped that the

outline may have been sufficient to indicate the very fundamental character of those problems.

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June, 1920.

[1] U. S. Geol. Survey, *Prof. Paper* **85C**, 1913 (35).

[2] See Lambert, *Journ. Wash. Acad. Sci.*, **10**, 1920 (122–143).

[3] In this connection it is important that the study of the composition and probable sources of the matter now being received by the earth, in the form of stony and metallic meteorites, be continued and extended.

THE IONIZATION OF STRONG ELECTROLYTES

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Communicated by J. Stieglitz, April 14, 1920

It is well known that in all branches of knowledge it is sometimes difficult to express certain thoughts with accuracy on account of the fact that the meaning of words changes. So in science there are periods when either the same words have a dissimilar meaning, or unlike words have the same meaning, to different workers. This sometimes becomes so marked that old hypotheses may seem to disappear, or new ones to appear, when only the significance of the technical terms has undergone a change. In other cases old terms may be used in new ways to suggest valuable new points of view.

The recent paper by A. A. Noyes and D. A. MacInnes in these PROCEEDINGS[1] which supports the idea of Sutherland,[2] Milner,[3] Ghosh,[4] and Bjerrum,[5] that strong electrolytes are completely *ionized* in solution, may be considered as belonging to the last category. They use the term ion in the sense of a charged particle, and not as a charged particle which is free to move in the electric field, which is the older meaning of the term as applied to solutions, though it must be admitted that their use of the word is in harmony with the meaning attached to it in recent papers in physics. However, it is not the terminology, but the idea it is apt to convey to workers in various fields, which is to be the subject of attention in this short note. The idea developed by Bragg, that a solid salt consists not of a collection of molecules, but of atoms, has led to the

hypothesis that such salts are collections of polar atoms, which are often called ions.

Let us suppose that in the gaseous state in a system there are sodium atoms in combination with chlorine atoms. These pairs are molecules, since, on account of their considerable relative distance from other particles, their electromagnetic fields are largely self contained. It is probable that if such molecules remain at temperatures at which the solid salt exists, the distance of the sodium from the chlorine atom, will not be very different from the similar distance in the solid state. On the basis of the nomenclature which considers that the solid salt consists of ions alone, it might be said that the gaseous molecule is also ionized (called intra-molecular ionization by Thomson), but still remains together as a single molecular particle, though it seems preferable to use the terms presented by Lewis and Bray, and consider the molecule as polar, and as made up of polar atoms. If many of these gas molecules pass into an aqueous system, then it is probable that the polar atoms in practically all of them, will separate to greater distances than that between unlike atoms in the gaseous molecules. In any event the sodium chloride particles would not persist as molecules, since their electromagnetic fields would interact with those of the solvent (this action is commonly known as hydration or solvation). If the terminology is such that the salt is considered as 100% ionized in the solid or gaseous state, then it would be absurd to say that it is less than totally ionized in solution. However, many of the pairs of positive and negative polar particles, are so close together that they are not free to move in the electric field, and so in the accepted sense of the term, there are un-ionized or non-ionized aggregates. On the basis of the complete ionization convention these may be called *bound*, and those which are far apart, *free* ions.

From this standpoint let us consider that there exist at any instant in the solution, all degrees of firmness of binding, from ions which are far apart and almost entirely free, down to those which are most closely bound, and therefore are not ions in the accepted sense. Suppose that the problem is assigned of determining the degree of separation or of dissociation of the electrolyte. It is obvious that according to the way in which the distinction is drawn between the dissociated and undissociated fractions, the calculated results will differ.

In order to prevent a confusion in terms in my own class work I have, for the past seven years used the expression, highly ionized salts, acids, and bases, are completely ionized or polarized in aqueous solution, but what may be considered the mean degree of separation or of dissociation, depends upon the method by which it is determined, since it is not to be expected that different measuring devices will draw the line of distinction at exactly the same place.

It is of interest in this connection to note that while in the papers previously cited the hypothesis is presented that the ionization is complete, and while according to the commonly accepted theory the ionization of such electrolytes, when uni-univalent, is about 0.85% at tenth-normal concentration, G. N. Lewis, and Lewis and Linhart,[6] have advanced the idea that the corresponding value is 68.4% (as calculated from the data of Hall and Harkins on sodium and potassium iodates). This they call the thermodynamic or corrected dissociation. It may, therefore be considered that such salts are 100% polarized or ionized, 85% electrolytically dissociated, and 68% thermodynamically dissociated. All of these conceptions are probably of use, and no one of them refers to the same feature of the phenomenon as any of the two others.

The confusion arises partly, as has been stated, from the attempt to consider a kinetic or a statistical problem as giving an average or mean which has no meaning from the standpoint of the condition in the solution, but has a considerable value as the basis for calculations by the use of empirical methods, as for example calculations of solubility or of electromotive force. Those who rally to the support of the complete ionization theory are doing so largely to do away with these mean values, and their endeavor is to introduce a statistical theory which shall be more closely descriptive of the phenomena. However, I wish to point out that to go over completely to this point of view at the present time, before the statistical theory has been worked out in an entirely satisfactory manner, is to lose some of the useful features of the older theory.

Only one of these features will be discussed. In 1911 I showed[7] that the effects of salts upon the solubility of unibivalent salts, are such as to almost inevitably lead to the conclusion that unibivalent salts ionize, or dissociate in steps, to form what were then called intermediate ions. According to Milner's general nomenclature these might be called associated ions. A calculation made by G. N. Lewis showed[8] that the transference and solubility results on this type of salts, could only be brought in harmony, on the assumption that the intermediate ion is so largely separated into its component ions, and that while it moves together under the influence of the electric field, it offers practically the same resistance to motion through the solution as if it were completely separated. Nevertheless, that there is such a union into an "intermediate" or associated, or possibly "complex" ion is clearly indicated by the phenomena. Noyes and MacInnes, though they do not consider this problem, seem to take the point of view that such relations can be accounted for by changes in the activities of the ions. However, the solubility relations of such salts could have been predicted from the standpoint of the intermediate-ion theory without any further knowledge of the relations other than those of the ordinary theory of solutions; but the activity concept, like other thermodynamic relations, is not specially fruitful in this particular sense,

though it is a powerful tool in many calculations where the premises are sufficiently known. In the end the complete ionization theory will undoubtedly be forced to consider in this case that the uni-univalent ion is often associated with the bivalent ion, and so will lead to the same conception as the older theory; but it may in the end be able to overcome the imperfections in the latter which are due to the fact that it ignores too greatly the kinetics of the phenomena, though it should be recognized that many of those who use the older theory are fully aware of the statistical features of the problem. In a certain sense it is probably nearer the truth to say that there is only one theory which is supported by the most advanced workers in the field, but that some of them pay attention to one set, and others to another set, of the features of the general theory, which will not be even approximately satisfactory until it is put much more completely into the form of partial differential equations.

Even though many physical chemists may dislike such a use of the term, it may be possible that the work ion will be used most largely in the sense in which it has been used by Noyes and MacInnes. In order to preserve some of the more valuable features of the older use of the term, in case this should occur, I wish to suggest that then a distinction be made, according to the suggestion made earlier in the paper, between ionization, and dissociation, the former term to be used whenever the particle becomes charged, the latter only when there is an obvious separation. Of course this does not surmount the difficulty that there are all degrees of separation, so there will be actually no such thing as a certain percentage dissociation. However, it may be said that the *apparent* percentage dissociation as determined by the conductivity or the thermodynamic method, has a definite value, which may, in certain cases, depend not only upon the experimental method employed, but also upon the additional hypotheses used in the calculation. What has been styled the unionized salt, may be called the undissociated salt. Intermediate or complex ions under the old nomenclature, may keep their older designations, or may be termed ion associates or complexes.

Unless those who speak of the complete ionization of all strong electrolytes do something of this sort, they will be plunged into difficulties if they attempt to use the energy concept (activity) to describe such phenomena as those in which the mercury and cadmium complexes seem, according to the commonly accepted view, to play an important part. They will have to explain also the slightly dissociated electrolytes, and those electrolytes in formic acid, which, although largely dissociated, follow the mass law, as has been found by Schlesinger.

It is always best when possible to devise new terms to express new ideas, since otherwise the point of view of all of the older literature is confused. It may be preferable, as has been suggested to the writer by Professor Stieglitz, to consider a gaseous salt molecule as made up of

polar atoms, and not of ions, and to use the term ions only in its historical sense, that is to designate particles which migrate in the electrical field.

[1] Noyes and MacInnes, these PROCEEDINGS, **5**, 1919; *J. Amer. Chem. Soc.*, **42**, 1920 (239).
[2] *London, Phil. Mag.*, Ser. 6, **14**, 1907 (3).
[3] *Ibid.*, **35**, 1918 (214, 354).
[4] *J. Chem. Soc.*, **113**, 1918 (449, 627).
[5] Bjerrum, *Zs. Elektrochem.*, **24**, 1918 (321).
[6] *J. Amer. Chem. Soc.*, **33**, 1911 (1807, 1827, 1836).
[7] *Ibid.*, **41**, 1919 (1951); **34**, 1912 (1631).
[8] *Ibid.*, **33**, 1911 (1864).

ON A CONDITION FOR HELMHOLTZ'S EQUATION SIMILAR TO LAMÉ'S

BY ARTHUR GORDON WEBSTER

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Communicated July 14, 1902

During the last thirty years the writer has been very much interested in the diffraction of sound, a subject suggested to him for theoretical treatment in 1888 by his teacher, the great von Helmholtz. Considering the great amount of paper spoiled in futile attempts to further the subject, the pessimistic view of Lord Rayleigh, and the amount of experimental results obtained by the writer, but not published, it seems proper, in accordance with a policy recently announced by the writer, to publish whatever he has in storage, however modest. A small attempt was made in a paper "On the Wave Potential of a Circular Line of Sources (*Proc. Amer. Acad. Arts Sci.*, December, 1911), an improvement on which has been recently made. The following paper is taken from a drawer, endorsed February 20, 1908, and like the other was written in the attempt to advance the theory of the megaphone.

The condition obtained by Lamé that a singly infinite family of surfaces shall be the equipotentials for some distribution is well known. It occurred to me to examine the condition that there may be a function V satisfying the differential equation investigated by von Helmholtz,

$$\Delta V + k^2 V = 0, \tag{1}$$

which comes from the wave equation

$$\frac{\partial^2 \varphi}{\partial t^2} = a^2 \Delta \varphi, \tag{2}$$

when we assume that φ contains a simple harmonic function of the time, the function V depending upon a single parameter q. If this is the case we have

$$\frac{dV}{dq}\Delta q + \frac{d^2V}{dq^2}h^2_q + k^2V = 0, \tag{3}$$

where Δ_q stands for the Laplacian and h_q for the gradient of q. Dividing by dV/dq we have

$$\Delta q + h^2_q \frac{d(\log dV/dq)}{dq} + \frac{k^2}{d(\log V)/dq} = 0. \tag{4}$$

Since V and its derivatives depend on q alone, we may write this

$$\Delta q + h^2_q \varphi(q) + \frac{k^2}{f(q)} = 0 \tag{5}$$

where

$$f(q) = \frac{d(\log V)}{dq}, \quad \log V = \int f(q)dq + C, \quad V = Ae^{\int f(q)dq}$$

$$\frac{dV}{dq} = Ae^{\int f(q)dq} f(q), \quad \log \frac{dV}{dq} = \log A + \int f(q)dq + \log f(q)$$

$$\varphi(q) = d\,\frac{\left(\log \frac{dV}{dq}\right)}{dq} = f(q) + \frac{f'(q)}{f(q)}.$$

Thus we may write (5).

$$\Delta q + h^2_q \left\{ f(q) + \frac{f'(q)}{f(q)} \right\} + \frac{k^2}{f(q)} = 0, \tag{6}$$

which is the required condition. This may be put in another form as follows: Multiplying by $f(q)/h^2_q$ we have

$$\frac{df}{dq} + f^2 + \frac{f\Delta q + k^2}{h^2_q} = 0. \tag{7}$$

Now although it is not necessary for either Δq or h^2_q to be functions of q alone, since the first two terms of (7) are such, the third term must be, say

$$\frac{f\Delta q + k^2}{h^2_q} = Q(q)f(q). \tag{8}$$

The equation

$$\frac{df}{dq} + f^2 + Qf = 0, \tag{9}$$

is not linear, but may be made so by substituting for f its reciprocal R. We then have

$$\frac{dR}{dq} - QR = 1 \tag{10}$$

of which the solution is

$$R = \frac{1}{f} = e^{\int Qdq}(B + \int e^{-\int Qdq}dq). \tag{11}$$

From (11) and (8) we accordingly obtain

$$\Delta q = Qh^2_q - k^2R = Qh^2_q - k^2e^{\int Pdq}(B + \int e^{-\int Qdq}dq), \tag{12}$$

which is the desired condition, containing one arbitrary constant and one arbitrary function, and reducing to Lamé's condition when $k = 0$.

It is easy to show that (12) is satisfied by a set of parallel planes, concentric spheres, or concentric circular cylinders, all these being parallel surfaces. It is not satisfied, as is the case with Laplace's equation, by a set of confocal ellipsoids. It was owing to the impossibility of finding other sets of surfaces to test the result, together with compunctions about Huygens' principle, that caused the writer to hold back the result. Recently, however, Somiliana has published a proof that these are the only sets of parallel surfaces that allow propagation according to Huygens' principle (*Atti Torino*, 54, (974–979). Sulle relaziono fra il principio di Huygens e l'ottica geometrica) in which he is obliged to use the formulae of differential geometry. I therefore venture to publish the above more simple result.

THE ABSORPTION OF X-RAYS BY CHEMICAL ELEMENTS OF HIGH ATOMIC NUMBERS

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Communicated August 19, 1920.

Introduction.—A critical absorption wave-length characteristic of a chemical element is a wave-length such that the element absorbs X-rays longer than the critical value less than it does X-rays shorter than that value. Each chemical element has one critical absorption wave-length associated with its K series of characteristic emission lines. Duane and Hu[1] have shown that in the K series of rhodium the critical absorption wave-length is about one-fourth of one per cent shorter than that of the shortest line (the γ line) in the K emission series. The authors[2] found that the critical absorption in the K series of tungsten has a wave-length of about one-half of one per cent shorter than that of the $K\gamma$ emission line. Since the other characteristic X-ray series have longer wave-lengths than those of the K series, the K critical absorption wave-length is the shortest X-ray wave-length now known to be characteristic of a chemical element.

In the research described in this note the authors have measured the critical absorption wave-lengths in the K series of most of the available chemical elements from tungsten to uranium, both inclusive. They used an ionization spectrometer, and examined spectra of the first, second and third orders. In 1918 Duane and Shimizu[3] measured four of these wave-lengths in spectra of the first order by the ionization method. Measurements with the same spectrometer of the K critical absorption had previously been made for most of the chemical elements down to man-

ganese (atomic number 25).[4] De Broglie,[5] and Siegbahn and Jönsson[6] have published measurements of the K critical absorption by means of photographic spectrometers for the chemical elements of high atomic numbers included in this research. They examined spectra of the first order. Columns 3 and 4 in the table contain the values of the wave-lengths they give.

Apparatus and Method.—The general method of making the measurements does not differ essentially from that employed by Duane and Shimizu.[3] We have been able, however, to apply a somewhat higher voltage to the X-ray tube than in the earlier researches. The X-ray tubes we have used do not withstand a *constant* difference of potential greater than about 115,000 volts. To produce X-rays shorter than the K series of uranium, however, requires a voltage considerably in excess of this figure. By encasing the arms of the X-ray tube and also those of the kenotrons belonging to the high tension generating plant in oil baths we have been able to excite the tube with approximately constant voltages up to about 140,000 volts. We estimated these voltages with an electrostatic voltmeter, which we calibrated by measuring the current from the generating plant through a series of coils of manganin wire having a total resistance of 6.043,000 ohms. As a check on the voltage measurements and on the constancy of the difference of potential we determined the short wave-length limit of the general X-ray spectrum and calculated the voltage by means of the quantum equation $Ve=h\nu$. Duane and Hunt[7] showed by experiments that this law holds for the limit of the spectrum, and their results have been verified by more recent researches.

Results of the Measurements.—On platting the currents in the ionization chamber against the angular positions of the table on the spectrometer that supports the crystal we obtain curves, examples of which appear in figures 1 and 2. The sharp drops, a, in these curves, represent the critical absorption due to the chemical element under investigation. A layer of matter containing this element is placed in the path of the X-rays between the X-ray tube and the spectrometer. The angular distance between the mid points in corresponding drops on the two sides of the zero gives us twice the glancing angle, θ, which must be substituted in the usual formula,

$$\lambda = 2d \sin \theta$$

to calculate the critical absorption wave-length. A correction for excentricity, amounting to about 27″ of arc, has to be subtracted from the values of θ measured on the graphs.

One pair of curves in figure 1 represents the critical absorption of uranium. These curves are of special interest, for they correspond to the shortest characteristic X-ray (of any chemical element) that has been discovered up to the present time. The centres of the drops can be

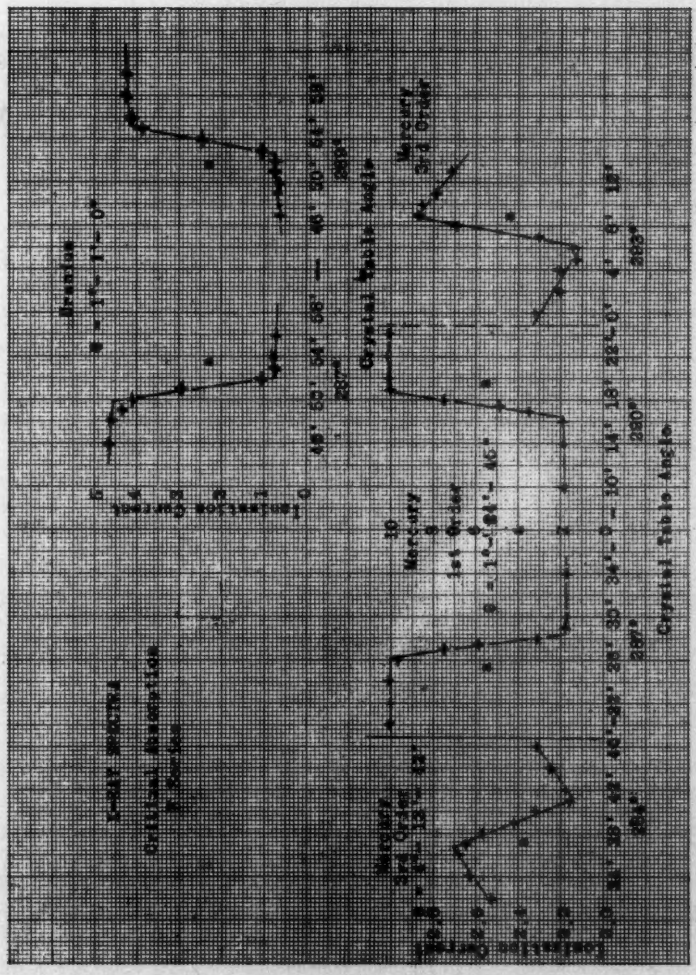

FIG. 1

estimated with reasonable certainty to within about 10″ of arc, and, as θ equals 61′ in this case, the error of precision amounts to less than one-third of one per cent.

The other curves in figure 1 and the curves in figure 2 belong to mercury and bismuth, respectively. One pair of curves represents the critical absorption of bismuth in the second order spectrum, and another pair, the critical absorption of mercury in the third order. The drops corresponding to the absorption in the second and third order spectra are very much smaller than in the first order, and for this reason it is

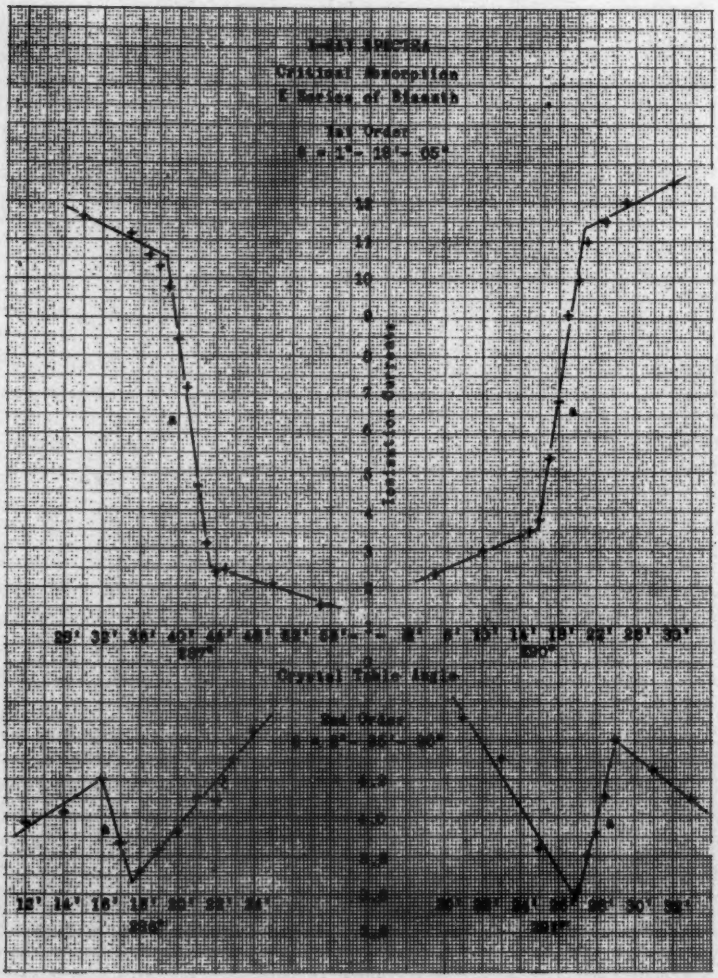

FIG. 2

more difficult to estimate their centers. The value of θ, however, is two or three times as large as in the first order.

Columns 5, 6 and 7 in table 1 contain the critical absorption wave-lengths estimated from the curves of figures 1 and 2 (and similar curves for the other chemical elements) in the spectra of the first, second and third orders respectively. Column 8 contains weighted mean values of the wave-lengths, column 9, the square roots of the wave-numbers, and

TABLE 1.

K CRITICAL ABSORPTION WAVE-LENGTHS, $\lambda \times 10^8$ Cm.

For Calcite $\lambda = 6.056 \times s$ in $\theta \times 10^{-8}$ Cm.

CHEMICAL ELEMENT	ATOMIC NUMBER	DE BROGLIE	SIEGBAHN JÖNSSON	AUTHORS' VALUES					
				1ST	2ND	3RD	AVERAGE	$\sqrt{1/\lambda} \times 10^{-4}$	DIF.
Tungsten*	74	.....	.....	.1783	.1781	.1780	.1781	2.370	
Platinum	78	.152	.1578	.1582	.....	.1581	.1581	2.515	.036
Gold	79	.149	.1524	.1537	.....	.1532	.1534	2.553	.038
Mercury	80	.146	.1479	.1493	.....	.1488	.1491	2.590	.037
Thallium	81	.142	.1427	.1448	.....	.1449	.1448	2.628	.038
Lead	82	.138	.1385	.1412	.1409	.....	.1410	2.663	.035
Bismuth	83	.133	.1346	.1375	.1369	.....	.1372	2.700	.037
Thorium	90	.....	.1127	.1139	.1124	.....	.1131	2.974	.039
Uranium	92	.....	.1048	.1075	.....	.....	.1075	3.051	.037

* The wave-lengths for tungsten were measured in coöperation with Dr. R. A. Patterson.

column 10, the difference, between these square roots calculated per unit increase in atomic number.

Discussion of the Results.—The differences tabulated in column 10 do not vary from their mean value more than one would expect from the experimental errors. This indicates that within the limits of error $\sqrt{1/\lambda}$ is a uniformly increasing function of the atomic number. The differences, however, are slightly larger than the corresponding differences between the square roots of the critical absorption wave-numbers measured in previous researches[3,4] for chemical elements of lower atomic numbers. This means that the square root of the critical absorption wave-number is not quite a linear function of the atomic number, which agrees with the results noted in the earlier papers.[3,4] The variation from the linear law, however, is too small to be clearly indicated in the range of values contained in table 1.

The wave-lengths measured in spectra of the second and third order (columns 6 and 7) appear to be, with one exception, smaller than the corresponding wave-lengths measured in the first order spectrum. The differences between the two sets of values about equal the errors of measurement. Similar differences between spectra of the various orders were observed first by Dr. Stenström[5] in his measurements of long X-rays by means of a photographic spectrometer. He attributed the effect to a slight refraction and dispersion of the X-rays by the reflecting crystal.

Our measurements in the second order spectrum for thorium were not

as satisfactory as the others. The generating plant had to be pushed to the limit of its capacity, and the fluctuations in the X-ray output were unusually large.

Our values of the wave-lengths are uniformly larger than those obtained by photographic methods. On the average these differences amount to between one and two per cent. The measurements by Siegbahn and Jönsson were made with an interesting spectrometer specially designed to eliminate errors due to the penetration of the X-rays into the reflecting crystal, etc. In an ordinary spectronometer, if the grazing angle θ, is determined by the position of the reflected beam of X-rays these errors may be either positive or negative, according to the position of the axis of rotation of the crystal with reference to what we have called the *effective* reflecting plane. Further, they may increase or decrease with the wave-length of the rays, and in limiting cases the size of the crystal would have an effect on them. Our method of using the spectrometer eliminates these errors, for we determine the grazing angle, θ, from the angle through which the crystal turns and not by the position of the reflected beam. The differences, therefore, between our values and those given by Siegbahn and Jönsson cannot be ascribed to errors of this kind. Possibly they may be due to differences in the manner of interpreting the experimental observations. If, for instance, measurements on a photographic plate were made from the point where the plate *begins* to get dark, the corresponding value of the critical absorption wave-length would be shorter than that given by our method, for we measure between points half way up the steep drops. The point where the photographic plate begins to darken corresponds to the bottom of a drop on our curves.

The wave-lengths contained in table 1 complete the series of measurements of the K critical absorption of the chemical elements that we have been making in our X-ray laboratory during the past few years.[3,4] We now have values of the K critical absorption wave-lengths for most of the chemical elements from manganese (atomic number 25) to uranium (atomic number 92) both inclusive. These wave-lengths have been measured by means of the same ionization spectrometer and with the same calcite crystal, and are therefore comparable with each other.

[1] Duane and Hu, *Physic. Rev., Ithaca, N. Y.*, Oct., 1918 (369).
[2] Duane and Stenström, *Washington, Nat. Acad. Proc.*, Aug., 1920.
[3] Duane and Shimizu, *Physic. Rev.*, Dec., 1919 (522).
[4] Blake and Duane, *Ibid.*, Dec., 1917, 697; and Duane and Hu, *Ibid.*, Dec., 1919 (516).
[5] De Broglie, *J. Physique, Paris*, May–June, 1916 (161).
[6] Siegbahn and Jönsson, *Physik. Zeits.*, 1919 (251).
[7] Duane and Hunt, *Physic. Rev.*, Aug., 1915 (166).
[8] Stenström, Doctor's *Dissertation*, Lund., 1919.

THE THOMSON EFFECT AND THERMAL CONDUCTION IN METALS

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The Thomson Effect.—In these PROCEEDINGS for March, 1920, is a paper in which I undertook to account for the Thomson effect values, as found by Bridgman for many metals, in accordance with the hypothesis of dual electric conduction.

Using data obtained by the methods of that paper, I was able, as I stated at the Washington meeting of the National Academy in April, 1920, to account quantitatively for the values of thermal conductivity found in a large number of metals at room temperature. I stated further, however, that the temperature-coefficient of thermal conductivity indicated by my calculations was far too large to accord with the obvious facts, and that a revision of my formulae was therefore necessary. This revision, which has now been made, enables me to deal pretty satisfactorily with thermal conduction, if my assumptions are granted. It involves important changes in most of the numerical tables in the paper referred to, but no great change in the method there followed.

The revision begins with equation (11), which is changed by the substitution of s for 2.5, so that it now stands

$$\lambda' = \lambda'_0 + s R T, \qquad (1)$$

where s is a constant, which varies from one metal to another but is always greater than 2.5. The λ' of this equation is the number of ergs required to separate one electron from its atomic union and leave it free, as a gas particle, within a metal; λ'_0 is a constant which may be different in different metals, and R is the gas constant for a single molecule. The assumption involved in this equation, like some others I have made, may seem improbable; but, as it does not, so far as I am aware, conflict with established facts or principles,[1] and as it serves my purpose fairly well, I have felt justified in adopting it provisionally.

As the total energy which a free electron possesses in virtue of its character as a monatomic gas molecule is $2.5\ RT$, the kinetic energy and the pv energy being taken together, and as I assume that this energy is acquired in the act of ionization, I have

$$\lambda' - 2.5\,RT, \text{ or } \lambda'_0 + (s - 2.5)RT,$$

as the number of ergs required for overcoming the attractions or repulsions which are operative in the act of freeing a single electron from an atom, under the conditions which prevail within a solid metal. The

supposed increase of this quantity with rise of temperature may be due to the expansion of the metal.

The meaning of P_a and P_f, symbols which I have used before and shall presently use again, is such that

$$P_f - P_a = \frac{1}{e}(\lambda'_o + (s - 2.5)RT), \qquad (2)$$

where e is the electron charge.

The revision leaves equations (12), (13), (17) of my previous paper unchanged, but in (14) it substitutes $(s + 1.5 - q)$ for $(4 - q)$ in the first term of the second member, and in (15) it puts $(2s + 1.5 - q)$ in place of $(6.5 - q)$.

Thermal Conduction.—In December, 1917, I proposed[2] the following theory: That thermal conduction in a metal is due to the convective action of a circulating electric current, free electrons moving down the temperature gradient and associated electrons moving up, with ionization at the hot part of the metal, involving absorption of heat, and reassociation at the cold end, involving the emission of heat. I now propose to go as far as I can at present in the way of submitting this theory to a quantitative test.

As in my previous papers, "hypothesis (*A*)" will mean the assumption that the purely mechanical tendency of the free-electron gas is toward equality of pressure throughout the metal. Under this hypothesis the condition of equilibrium in a detached metal bar having an established and permanent temperature gradient is expressed by the equation

$$k_f\left(\frac{1}{ne}\frac{dp}{dl} + \frac{dP}{dl} + \frac{dP_f}{dl}\right) = -k_a\left(\frac{dP}{dl} + \frac{dP_a}{dl}\right). \qquad (3)$$

This comes from equation (1) of my paper in these Proceedings for April, 1918, by substitution for μ according to equation (2) of that paper, which *should be*

$$\mu = k_f \frac{I}{Gne^2}.$$

The first member of (3) as just given is the strength of the free-electron current, down the temperature gradient, and the second member is the strength of the associated-electron current, up the temperature gradient, per unit cross-section of the bar. We can rewrite (3) as follows, taking, for simplicity, the case in which $(dT \div dl) = 1$:

$$k_f\left(\frac{1}{ne}\frac{dp}{dT} + \frac{dP}{dT} + \frac{dP_f}{dT}\right) = -k_a\left(\frac{dP}{dT} + \frac{dP_a}{dT}\right). \qquad (4)$$

The term $(dP_f \div dT)$ expresses a possible differential attraction of the unequally heated metal for the free electrons, tending to move them down the temperature gradient, something distinct from the force due to the gradient of potential $(dP \div dT)$, which the gas-pressure drift

of the free electrons produces. The term $(dP_a \div dT)$ has a corresponding meaning with regard to the associated electrons.

The relation between P_f and P_a being expressed by equation (2), we must suppose at least one of the ratios $(dP_f \div dT)$ and $(dP_a \div dT)$ to have a finite value. I shall assume the value of $(dP_a \div dT)$ to be negligible, and therefore we shall have, from equation (2),

$$\frac{dP_f}{dT} = \frac{R}{e}.(s - 2.5). \tag{5}$$

Since

$$p = n\,RT, \tag{6}$$

and, according to a previous assumption,

$$n = zT^q, \tag{7}$$

where z and q are constants, we have

$$\frac{1}{n}\frac{dp}{dT} = R\left(1 + \frac{T}{dT}\frac{dn}{n}\right) = R\,(1 + q). \tag{8}$$

Substituting from (5) and (8) in (4) and taking $(dP_a \div dT)$ as zero, we get

$$k_f\left(\frac{R}{e}(s + q - 1.5) + \frac{dP}{dT}\right) = -k_a\frac{dP}{dT}. \tag{9}$$

According to my conception of thermal conduction, we have only to multiply either side of this equation by λ, the amount of energy required to free $(1 \div e)$ electrons within the metal, to get the value of θ, the thermal conductivity of the metal. That is, we can write

$$\theta = -\lambda k_a\frac{dP}{dT}. \text{ ergs/cm., sec., deg. } C. \tag{10}$$

We must now deal with the individual factors in the second member of this equation. From (9), remembering that $k = k_a + k_f$, we get

$$\frac{dP}{dT} = -\frac{R}{e}\frac{k_f}{k}(s + q - 1.5), \tag{11}$$

and λ is $(1 \div e)$ times the λ' of equation (1).

Hence we have

$$\theta = \frac{R^2}{e^2}\left(\frac{\lambda'_o}{R} + sT\right)k_a\frac{k_f}{k}(s + q - 1.5). \tag{12}$$

Of course, since k is known, k_a is known if k_f is known. Substantially, my method of procedure with each metal is to find by trial values of λ_o', s, $(k_f \div k)$, and q, that will in combination account for Bridgman's value of σ, the Thomson effect, and then from the many combinations that will do this to find the smaller number that will account also for the known value of θ. Ultimately I must undertake to find from the combinations that meet these two tests the ones that will account for or be consistent with the Peltier effect, but I shall not do that in this paper.

In the following tables, as in the corresponding tables of my March paper, the metals are divided into four groups, according to their values of K_1 and K_2 in the equation $\sigma' = (K_1 + K_2 t)T$. Two alloys also, constantan and manganin, are dealt with according to the data given for them by Bridgman. In finding k_{100}, the electrical conductivity at 100° C., from k_0, the conductivity at 0° C., I have used Bridgman's temperature coefficients; but he, working with very fine wires, did not determine the absolute conductivity at any temperature. The values of θ_0 and θ_{100} given in the first line beneath the name of any metal are derived from the work of Jaeger and Diessel-horst, except in those cases where a footnote is referred to. They are expressed in cal./cm., sec., deg. C. C, C_1 and C_2 are constants in the equation

$$(K_f \div K) = C + C_1 t + C_2 t^2.$$

The δ_c of these tables is the constant part of the "ionizing potential," expressed in volts; δ_0[3] is the total ionizing potential at 0° C., and δ_{100} is the total ionizing potential at 100° C.

The letter y indicates the estimated percentage of atoms which are ionized at 0° C. The method of estimation is given later.

First Group: Metals for which $K_1 > 0$ and $K_2 = 0$.

TABLE 1: COBALT

$k_0 = 102 \times 10^{-6}$; $k_{100} = 74.7 \times 10^{-6}$; $\theta_0 = 0.161$;[1] $\theta_{100} = 0.164$;[1] $(\theta_0 \div \theta_{100}) = \lambda.098$

$\sigma = 7.8\ T$ ergs/deg. C.

If $q = 1.55$ and $s = 4$ and $\lambda'_0 = 42\ R$, $C_1 = 230 \times 10^{-6}$ and $C = 0.256$

$\left(\frac{kf}{k}\right)_{100}$	γ	δ_c	δ_0	δ_{100}	$\left(\frac{dP}{dT}\right)_0$	$\left(\frac{dP}{dT}\right)_{100}$	$(k_a)_0 \times 10^8$	$(ka)_{100} \times 10^8$	θ_0	θ_{100}	$\frac{\theta_0}{\theta_{100}}$
0.279	6.7%	0.004	0.098	0.132	−8920	−9720	76.0	54.0	0.157	0.165	0.95

[1] Lacking observed values of θ_0 and θ_{100} for cobalt, I have estimated the values here given by use of the Wiedemann-Franz ratio (average for 13 metals) applied to k_0 and k_{100}.

TABLE 2: NICKEL

$k_0 = 93 \times 10^{-6}$; $k_{100} = 62.5 \times 10^{-6}$; $\theta_0 = 0.143$; $\theta_{100} = 0.138$; $(\theta_0 \div \theta) = 1.04$

$\sigma = 3.56\ T$ ergs/deg. C.

If $q = 1.55$ and $s = 4$ and $\lambda'_0 = 100\ R$, $C_1 = 105 \times 10^{-6}$ and $C = 0.239$

0.250	5.7%	0.009	0.103	0.137	−8330	−8710	70.8	46.9	0.144	0.133	1.08

TABLE 3: PALLADIUM

$k_0 = 100 \times 10^{-6}$; $k_{100} = 75.9 \times 10^{-6}$; $\theta_0 = 0.165$; $\theta_{100} = 0.182$; $(\theta_0 \div \theta_{100}) = 0.91$

$\sigma = 3.52\ T$ ergs/deg. C.

If $q = 1.6$ and $s = 6$ and $\lambda'_0 = 114\ R$, $C_1 = 69.8 \times 10^{-6}$ and $C = 0.0987$

0.106	3.2%	0.010	0.151	0.203	−5180	−5550	90.1	67.9	0.168	0.182	0.92

TABLE 4: PLATINUM

$k_0 = 110 \times 10^{-6}$; $k_{100} = 79.3 \times 10^{-6}$; $\theta_0 = 0.163$; $\theta_{100} = 0.173$; $(\theta_0 \div \theta_{100}) = 0.95$
$\sigma = 2.67\ T$ ergs/deg. C.
If $q = 1.6$ and $s = 7$ and $\lambda'_0 = 117\ R$, $C_1 = 45.2 \times 10^{-6}$ and $C = 0.0652$

$\left(\frac{kf}{k}\right)_{100}$	γ	δ_c	δ_0	δ_{100}	$\left(\frac{dP}{dT}\right)_0$	$\left(\frac{dP}{dT}\right)_{100}$	$(k_a)_0 \times 10^6$	$(k_a)_{100} \times 10^6$	θ_0	θ_{100}	$\frac{\theta_0}{\theta_{100}}$
0.070	2.3%	0.01	0.174	0.235	−3980	−4260	103	74	0.170	0.175	0.97

TABLE 5: TIN

$k_0 = 105 \times 10^{-6}$; $k_{100} = 72.6 \times 10^{-6}$; $\theta_0 = 0.157$; $\theta_{100} = 0.145$; $(\theta_0 \div \theta_{100}) = 1.08$
$\sigma = 0.134\ T$ ergs/deg. C.
If $q = 1.51$ and $s = 8$ and $\lambda'_0 = 234\ R$, $C_1 = 1.97 \times 10^{-6}$ and $C = 0.0466$

0.047	2.3%	0.02	0.208	0.277	−3210	−3220	100	69.2	0.159	0.147	1.08

TABLE 6: ZINC

$k_0 = 180 \times 10^{-6}$; $k_{100} = 127 \times 10^{-6}$; $\theta_0 = 0.266$; $\theta_{100} = 0.262$; $(\theta_0 \div \theta_{100}) = 1.02$
$\sigma = 0.99\ T$ ergs/deg. C.
If $q = 1.55$ and $s = 10$ and $\lambda'_0 = 117\ R$, $C_1 = 11.64 \times 10^{-6}$ and $C = 0.0304$

0.032	1.6%	0.01	0.245	0.331	−2630	−2730	175	123	0.268	0.265	1.01

Second Group: Metals for which $K_1 < 0$ and $K_2 = 0$.

TABLE 7: BISMUTH

$k_0 = 9.3 \times 10^{-6}$; $k_{100} = 646 \times 10^{-6}$; $\theta_0 = 0.0201$; $\theta_{100} = 0.0161$; $(\theta_0 \div \theta_{100}) = 1.25$
$\sigma = -3.2\ T$ ergs/deg. C.
If $q = 1.25$ and $s = 2.8$ and $\lambda'_0 = 1170\ R$, $C_1 = -122.6 \times 10^{-6}$ and $C = 0.540$

0.528	2.9%	0.10	0.166	0.190	−11800	−11600	4.28	3.05	0.0200	0.0160	1.25

CADMIUM

$k_0 = 160 \times 10^{-6}$; $k_{100} = 112.4 \times 10^{-6}$; $\theta_0 = 0.223$; $\theta_{100} = 0.216$; $(\theta_0 \div \theta_{100}) = 1.03$
$\sigma = -32.4\ T$ ergs/deg. C.

It is easy to find values of q, s and λ'_0 that will give the right value for θ_0; but the value of θ_{100} that results is much too small. I cannot deal satisfactorily with cadmium at present, owing to the very large negative value that Bridgman finds for its σ, a value which is about ten times as great as those deducible from the work of various other investigators who have studied the thermo-electric behavior of this metal. See, for example, a paper by Dewar and Fleming, *Phil. Mag.*, Vol. 40, 5th Series, 1895. It is possible that Bridgman has made some mistake in this case.

TABLE 8: COPPER

$k_0 = 650 \times 10^{-6}$; $k_{100} = 455 \times 10^{-6}$; $\theta_0 = 0.920$; $\theta_{100} = 0.908$; $(\theta_0 \div \theta_{100}) = 1.01$
$\sigma = -0.966\ T$ ergs/deg. C.
If $q = 1.49$ and $s = 9$ and $\lambda'_0 = 3.24\ R$, $C_1 = -12.54 \times 10^{-6}$ and $C = 0.0372$

$\left(\frac{kf}{k}\right)_{100}$	γ	δ_c	δ_0	δ_{100}	$\left(\frac{dP}{dT}\right)_0$	$\left(\frac{dP}{dT}\right)_{100}$	$(k_a)_0 \times 10^6$	$(k_a)_{100} \times 10^6$	θ_0	θ_{100}	$\frac{\theta_0}{\theta_{100}}$
0.036	6.4%	0.003	0.214	0.292	−2880	−2780	626	439	0.918	0.850	1.08

TABLE 9: MAGNESIUM

$k_0 = 240 \times 10^{-6}$; $k_{100} = ?$; mean θ between 0° and 100° = 0.376
$\sigma = -0.008\ T$ ergs./deg. C.
If $q = 1.499$ and $s = 6.5$ and $\lambda'_0 = 460\ R$, $C_1 = -14.4 \times 10^{-6}$ and $C = 0.066$

0.066	12%	0.039	0.192	0.248	−3700	−3700	224	?	0.379	?	?

TABLE 10: SILVER

$k_0 = 670 \times 10^{-6}$; $k_{100} = 476 \times 10^{-6}$; $\theta_0 = 1.01$; $\theta_{100} = 0.99$; $(\theta_0 \div \theta_{100}) = 1.02$
$\sigma = -0.864\ T$ ergs/deg. C.
If $q = 1.49$ and $s = 11.5$ and $\lambda'_0 = 30\ R$, $C_1 = -8.8 \times 10^{-6}$ and $C = 0.0240$

0.023	5.5%	0.003	0.272	0.370	−2380	−2280	655	466	1.01	0.94	1.08

TABLE 11: TUNGSTEN

$k_0 = 140 \times 10^{-6}$; $k_{100} = 106 \times 10^{-6}$; $\theta_{18} = 0.35$;[1] $\theta_{100} = ?$
$\sigma = -3.41\ T$ ergs/deg. C.
If $q = 1.49$ and $s = 9$ and $\lambda'_0 = 18.4\ R$, $C_1 = -44.4 \times 10^{-6}$ and $C = 0.0696$

0.065	3.2%	0.002	0.213	0.290	−5380	−5030	130	99	0.355	0.344	1.03

[1] Value found by Coolidge.

Third Group: Metals for which $K_1 > 0$ and $K_2 > 0$.

TABLE 12: IRON

$k_0 = 78 \times 10^{-6}$; $k_{100} = 48 \times 10^{-6}$; $\theta_0 = 0.163$; $\theta_{100} = 0.151$; $(\theta_0 \div \theta_{100}) = 1.08$
$\sigma = (1.78\ T + 6.04\ tT)$ ergs/deg. C.
If $q = 1.205$ and $s = 11$ and $\lambda'_0 = 117\ R$, $C_2 = 26.9 \times 10^{-6}$, $C_1 = 14.8 \times 10^{-6}$, $C = 0.0364$

0.041	1.2%	0.01	0.268	0.363	−3350	−3740	75.2	46.2	0.161	0.149	1.08

TABLE 13: THALLIUM

$k_0 = 57 \times 10^{-6}$; $k_{100} = 43.9 \times 10^{-6}$; $\theta_0 = 0.090(?)$;[1] $\theta_{100} = 0.096(?)$;[1] $(\theta_0 \div \theta_{100}) = 0.94(?)$
$\sigma = (0.268\ T \times 0.00336\ tT)$ ergs/deg. C.
If $q = 1.48$ and $s = 7$ and $\lambda'_0 = 117\ R$, $C_2 = 2.80 \times 10^{-4}$, $C_1 = 3.56 \times 12^{-6}$, $C = 0.0676$

0.068	1.9%	0.01	0.174	0.236	−4060	−4100	53.1	41.0	0.089	0.094	0.95

[1] See footnote under table 1, cobalt

TABLE 14: ALUMINIUM

Fourth Group: Metals for which $K_1 < 0$ and $K_2 > 0$

$k_0 = 380 \times 10^{-6}$; $k_{100} = 265 \times 10^{-6}$; $\theta^\circ = 0.477$; $\theta_{100} = 0.492$; $(\theta^\circ \div \theta_{100}) = 0.97$[1]

$\sigma = (-0.016\ T + 0.006\ tT)$ ergs/deg. C.

If $q = 1.45$ and $s = 5.5$ and $\lambda'_0 = 125\ R$, $C_2 = 6.35 \times 10^{-8}$, $C_1 = -3.04 \times 10^{-4}$, $C = 0.0886$

$\left(\frac{kf}{k}\right)_{100}$	γ	δ_C	δ_0	δ_{100}	$\left(\frac{dP}{dT}\right)_0$	$\left(\frac{dP}{dT}\right)_{100}$	$(ka)_0 \times 10^8$	$(ka)_{100} \times 10^8$	θ_0	θ_{100}	$\frac{\theta_0}{\theta_{100}}$
0.089	11%	0.011	0.140	0.187	−4150	−4170	346	241	0.479	0.449	1.07

[1] Lees found $\theta_{18} = 0.524$ in this metal; but apparently he did not determine θ_{100}.

TABLE 15: GOLD

$k_0 = 490 \times 10^{-6}$; $k_{100} = 351 \times 10^{-6}$; $\theta_0 = 0.699$; $\theta_{100} = 0.703$; $(\theta_0 \div \theta_{100}) = 0.994$

$\sigma = (-0.934\ T + 0.001\ tT)$

If $q = 1.495$ and $s = 8$ and $\lambda'_0 = 15\ R$, $C_2 = 0.73 \times 10^{-8}$, $C_1 = -13.7 \times 10^{-6}$, $C = 0.0486$

0.047	8%	0.002	0.190	0.259	−3340	−3290	466	335	0.705	0.672	1.05

TABLE 16: MOLYBDENUM

$k_0 = 274 \times 10^{-6}$; $k_{100} = 191 \times 10^{-6}$, $\theta_0 = 0.43(?)$;[1] $\theta_{100} = 0.42(?)$;[1] $(\theta_0 \div \theta_{100}) = 1.02(?)$

$\sigma = (-4.33\ T + 0.015\ tT)$ ergs/deg. C.

If $q = 1.3$ and $s = 10$ and $\lambda'_0 = 110\ R$, $C_2 = 8.7 \times 10^{-8}$, $C_1 = -51 \times 10^{-6}$, $C = 0.0338$

0.0296	2.8%	0.009	0.244	0.330	−2850	−2490	265	185	0.439	0.363	1.21

[1] See footnote under table 1, cobalt.

ALLOYS

TABLE 17: CONSTANTAN

$k_0 = 20.4 \times 10^{-6}$; $k_{100} = 20.4 \times 10^{-6}$, $\theta_0 = 0.052$; $\theta_{100} = 0.064$; $(\theta_0 \div \theta_{100}) = 0.81$

$\sigma = 7.94\ T$ ergs/deg. C.

If $q = 1.55$ and $s = 5$ and $\lambda'_0 = 90\ R$, $C_1 = 188 \times 10^{-6}$ and $C = 0.288$

0.306[1]	1%	0.008	0.126	0.169	−12500	−13300	14.5	14.2	0.054	0.076	0.71

[1] Very roughly estimated from the values of γ given for copper and nickel.

TABLE 18: MANGANIN

$k_0 = 23.8 \times 10^{-6}$; $k_{100} = 23.8 \times 10^{-6}$; $\theta_0 = 0.051$; $\theta_{100} = 0.063$; $(\theta_0 \div \theta_{100}) = 0.81$

$\sigma = (-0.828\ T - 0.00672\ tT)$ ergs/deg. C.

If $q = 1.51$ and $s = 4.5$ and $\lambda'_0 = 90\ R$, $C_2 = -8.74 \times 10^{-8}$, $C_1 = -18.0 \times 10^{-6}$

$C = 0.27$

0.267	1%[1]	0.008	0.113	0.152	−10500	−10400	17.4	17.5	0.049	0.066	0.75

[1] See footnote under table 17.

Comments.—There is a curious tenacity in the value of $(\theta_o \div \theta_{100})$ for any metal, as found by the method of this paper. Thus in no case except that of Co, where a great range of $(k_f \div k)$ was tried, did the value of $(\theta_o \div \theta_{100})$ change from >1 to <1 with any variations of q, s and λ'_o. Out of seventeen cases, including those of constantin and manganin, the value of $(\theta_o \div \theta_{100})$, as found by experiment or calculated by the use of the Wiedemann-Franz ratio, is in nine greater than 1 and in eight less than 1. In only two of these cases, those of aluminium and gold, does the method of this paper fail to give a value of $(\theta_o \div \theta_{100})$ on the right side of 1.

In nine of these seventeen cases, Bi, Co, Fe, Ni, Pd, Pt, Sn, Tl, Zn, it has been found practicable to get a combination of q, s and λ'_o that will account for Bridgman's σ and give the desired values of both θ_o and θ_{100} to any reasonable degree of accuracy. In all of these nine cases, except that of Bi, the value of K_1 in the equation $\sigma = K_1 T + K_2 tT$ is positive.

In six of the other eight cases the value of $(\theta_o \div \theta_{100})$ here found is larger than the one found by experiment or indicated by the *W-F* rule. In four cases, Ag, Al, Au, Cu, the discrepancy is about 6–10%; in one, that of Mo, it may be considerably greater, but this is a *W-F* case; in only one, that of Cd, is the disagreement hopelessly great. In constantin and in manganin the calculated $(\theta_o \div \theta_{100})$ is less than the observed.

The value of y, the percentage of ionized atoms within a metal, has been estimated by means of the formula[1]

$$k_f = \frac{n\,c\,l\,e^2}{4\,R\,T}.$$

When T is taken as 273, we have, approximately,

$$k_f = 1.91 \times 10^{-20}\, n\, l = 1.91 \times 10^{-20} \gamma v l,$$

or

$$\gamma = 5.24 \times 10^{-19}\, (k_f \div vl),$$

where v is the number of atoms of the metal per cu. cm. and l is the length of the "mean free path" of the electrons among the atoms.

Concerning the value of l, I have been and am still much in doubt. Adams and Chapman, in a study of the Corbino effect,[4] estimated it to be in copper not far from 3×10^{-7} cm., which is about 13 times the distance from centre to centre of adjacent atoms, if a cubical arrangement of the atoms is assumed. Compton, K. T., and Ross[5] concluded that a photo-electrically excited electron may move about 2.67×10^{-7} cm. in platinum, about 11 times the centre to centre distance of the atoms, and approximately 5×10^{-7} cm. in gold, nearly 20 times the atomic distance. I have taken l as 10 times the atomic distance, thus putting $l = (10 \div v^{\frac{1}{3}})$ and making

$$y = 0.52 \times 10^{19} \times k_f \div v^{\frac{2}{3}}.$$

A much smaller estimate of l than the one I have used would be disastrous for my speculations, as it would give large values of γ and thus in-

troduce an insuperable difficulty regarding specific heat. Even with the assumption that I have made as to the length of l, the influence of the free electrons, and still more the influence of ionization with increase of temperature, on the specific heat, presents a rather serious question. Thus, to take what is probably the most unfavorable case set forth in the preceding tables, if in Mg $q = 1.5$ and $s = 6.5$ and $\lambda'_0 = 460\ R$ and $\gamma = 12\%$, we have, as the heat absorbed by the free electrons in the rise of 1 gm. of Mg from 0 °C. to 1° C., 0.025 cal., while the heat required by the accompanying ionization is 0.124 cal., a total of 0.149 cal., which is rather more than half the total specific heat of Mg at 0° C. It would be easy, however, to choose values of q, s and λ'_0 for Mg which would serve the purpose of this paper while affecting the specific heat less. Moreover, it is possible that l may be greater than I have taken it to be.

Table 17 of my paper in these *Proceedings* for March, 1920, now requires revision, Al, Fe, Mo and Tl going into Section (A); but the general testimony of the table remains unchanged, the *mean* of the last column being now -1.50% for Section (A) and $+2.47\%$ for Section (B).

It remains to be seen whether the theory I am developing can deal successfully with the Peltier effect. A preliminary examination, already made, of this matter is encouraging.

[1] Equipartition of energy, as holding for electrons *within* an atom, cannot be regarded as an established principle.

[2] *Physic. Rev., Ithaca*, April, 1918 (329).

[3] In my previous paper δ_0 had a different meaning.

[4] *London, Phil. Mag.*, **28**, 1914 (692–702).

[5] *Physic Rev., Ithaca*, May 1919 (374–391).

MOTION ON A SURFACE FOR ANY POSITIONAL FIELD OF FORCE

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1. The first part of the paper presents a study of the geometric properties of the system of trajectories generated by the motion of a particle on any constraining surface (spread of two dimensions) under any positional field of force. The complete characteristic properties are derived.[1] Starting at any point on the surface in a given direction, and with a given speed, a unique trajectory is generated. The complete system of trajectories forms a triply infinite system of curves corresponding uniquely to a given field of force. The five geometric properties stated in section 2 are characteristic of the system of trajectories, and any triply infinite system of curves on a surface possessing these five properties may be considered as generated by the motion of a particle in a unique field of force.

An additional property serves to characterize the motion when the field of force is conservative.

Another part of the paper presents briefly an analogous study for certain other classes of triply infinite systems of curves on a surface, in particular, brachistochrones and catenaries in a conservative field of force. For all such systems five characteristic properties (sec. 3) differing but slightly from those for trajectories are derived.

2. The motion of a particle on a surface

$$x = x(u,v),\ y = y(u,v),\ z = z(u,v)$$

may be most simply expressed by the Lagrangian equations

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{u}}\right) - \frac{\partial T}{\partial u} = \phi,\ \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{v}}\right) - \frac{\partial T}{\partial v} = \psi,$$

where T is the kinetic energy, and ϕ, ψ are the components of the force given as functions of the coördinates[2] u,v. The differential equation of such a system found by eliminating the time has the form

$$v'' = A + Bv' + Cv'^2,$$

where A,B,C are special functions of u,v,v'.

The following five characteristic properties of dynamical trajectories under any positional field of force are derived by studying the differential equation of such a system.

I. The ∞^1 curves passing through a given point O in a given direction ξ have associated with them their orthogonal projections in the tangent plane to the surface at O. The locus of the foci of the osculating parabolas of the associate system is a bicircular quartic with O as node, and the straight line in the direction ξ both as tangent line to the quartic and also as one of the asymptotes to the hyperbola which is the inverse of the quartic with respect to the given point O.

II. The focal locus or bicircular quartic associated by Property I with each direction ξ through a point O, is such that the tangent line to the quartic in the direction ξ bisects the angle between the force vector through O and the second tangent to the quartic.

III. Through every point O on the surface and in every direction ξ through that point, there passes *one* curve of the system which hyperosculates its corresponding geodesic circle of curvature. The locus of the centers of geodesic curvature of the ∞^1 hyperosculating trajectories which pass through O is a conic passing through O in the direction of the force vector.

IV. The points of the surface and the directions of the force vectors through these form a set of differential elements defining a simple system of ∞^1 curves on the surface, viz., the lines of force. At any point O, the geodesic curvature of the line of force through O is equal to three times the geodesic curvature of the hyperosculating trajectory of Property III which passes through O in the same direction.

V. Construct any isothermal net on the surface. At any point O, this net determines two orthogonal directions in which there pass two isothermal curves of the net and two hyperosculating curves of Property III. If ρ_1, ρ_2, R_1, R_2 are the radii of geodesic curvature of these four curves, s_1, s_2, the arc lengths along the isothermal curves, and ω, the tangent of the angle between the force vector and the isothermal curve with arc s_2, then, as we move along the surface from O, these quantities vary so as to satisfy the relation

$$\frac{\partial}{\partial s_2}\left(\frac{1}{\kappa_1}\right) - \frac{\partial}{\partial s_1}\left(\frac{1}{\kappa_2}\right) - \frac{1}{\rho_1\kappa_1} + \frac{1}{\rho_2\kappa_2} - \frac{\partial^2(\log\omega)}{\partial s_1 \partial s_2} = 0,$$

where

$$\frac{1}{\kappa_1} = \omega\left(\frac{1}{\rho_1} - \frac{3}{R_1}\right), \quad \frac{1}{\kappa_2} = \frac{1}{\omega}\left(\frac{1}{\rho_2} - \frac{3}{R_2}\right).$$

If, in these five properties, we replace the force vector through a point by the tangent line to the conic of property III, we may state the theorem:

In order that a triply infinite system of curves (∞^1 in each direction through each point) on a surface may be identified with a system of dynamical trajectories under any positional field of force, the given system must possess properties I, II, III, IV, V.

When the surface is a developable surface, the bicircular quartic of property I reduces to a circle passing through the given point.

If the field of force is conservative, the conic of property III reduces to a rectangular hyperbola.

3. If we consider the motion of a particle on a surface in a conservative field of force from one position P_0 to another P_1 with the sum of the kinetic and potential energies equal to a given constant, h, certain systems of ∞^2 curves are defined by

$$\int_{(P_0)}^{(P_1)} \sqrt{W + h}^{\,n}\, ds = \text{minimum},$$

where W is the work function (negative potential) and ds is the element of arc length. Among such systems, called "n" systems, we may mention *dynamical trajectories* ($n = 2$), *brachistochrones* ($n = -2$), and *catenaries* ($n = 1$). We may find the differential equation of the ∞^2 curves of an "n" system by the methods of the calculus of variation.[3] If, by differentiation, we eliminate the constant of energy, h, from this equation, we shall get the differential equation of the ∞^3 curves of a complete "n" system. This equation has the form

$$v''' = A + Bv'' + Cv''^2,$$

where A, B, C are special function of u, v, v', and involve n as a parameter.

The following five characteristic properties of an "n" system on a surface are derived by studying the differential equation of such a system.

I'. The "n" system possesses property I of section 2.

II′. For an "n" system, the focal locus or bicircular quartic associated by property I′ with each direction ξ through a point O, is such that the tangent of the angle which the tangent line to the quartic in the direction ξ makes with the force vector is to the tangent of the angle which the second tangent line to the quartic makes with the first tangent line as $n + 1$ is to 3.

III′. The "n" system possesses property III of section 2. The conic described in this property is a rectangular hyperbola.

IV′. For an "n" system, at any point O on the surface, the geodesic curvature of the line of force through O is equal to $(n + 1)$ times the geodesic curvature of the hyperosculating curve of property III′ which passes through O in the same direction.

V′. For an "n" system, as we move along the surface from a point O, the quantities ρ_1, ρ_2, R_1, R_2, ω, s_1, s_2, defined in property V of section 2, vary so as to satisfy the relation

$$\frac{\partial}{\partial s_2}\left(\frac{1}{\kappa_1}\right) - \frac{\partial}{\partial s_1}\left(\frac{1}{\kappa_2}\right), \quad -\frac{1}{\rho_1\kappa_1} + \frac{1}{\rho_2\kappa_2} - \frac{\partial^2(\log\omega)}{\partial s_1\,\partial s_2} = 0,$$

where

$$\frac{1}{\kappa_1} = \omega\left(\frac{1}{\rho_1} - \frac{n+1}{R_1}\right), \quad \frac{1}{\kappa_2} = \frac{1}{\omega}\left(\frac{1}{\rho_2} - \frac{n+1}{R_2}\right).$$

If, in these five properties, we replace the direction of the force vector through a point by the direction of the tangent line to the rectangular hyperbola of property III′, we may state the theorem:

In order that a triply infinite system of curves on a surface may be identified with an "n" system, the given system must possess properties I′, II′, III′, IV′, V′.

[1] This is a summary of a long memoir offered for publication to the *Ann. Math., Princeton, N. J.* For a study of the corresponding problem in a plane or in ordinary 3-space, see Kasner, E., *Trans. Amer. Math. Soc., New York*, **7**, 1906 (401–424); also *Ibid.*, **8**, 1907 (135–158). For a summary of these two papers, see *Princeton Colloquium Lectures*, Chap. I.

[2] Dots refer to total derivatives with respect to the time t, and primes refer to total derivatives with respect to the parameter u.

[3] For a characterization of the ∞^2 curves of a natural system on a surface, defined by $\int F\,ds = \text{minimum}$, where F is a function of the coördinates only, see Lipka, J., *Ann. Math., Princeton, N. J.*, **15**, 1913 (71–77).